

Test #3 Review

Webwork - Nonhomogeneous Linear Problem 1

$$\star y'' - 5y' + 4y = 22 - 8t$$

(a) state characteristic polynomial:

$$r^2 - 5r + 4$$

$$(r-4)(r-1)$$

$$r = 1, 4 \quad (\text{case 1})$$

$$y_1 = e^x \quad y_2 = e^{4x} \quad \leftarrow \text{roots for assoc. homog. eqn.}$$

$$y = c_1 e^x + c_2 e^{4x} \quad \leftarrow \text{general sol'n for homog. eqn.}$$

Let's try $y_p = At + B$

$$y_p' = A$$

$$y_p'' = 0$$

Plug in to \star

$$0 - 5(A) + 4(At + B) = 22 - 8t$$

$$-5A + 4At + 4B = 22 - 8t$$

$$(4B - 5A) + 4At = 22 - 8t$$

$$4A = -8$$

$$\boxed{A = -2}$$

$$4B - 5A = 22$$

$$4B + 10 = 22$$

$$4B = 12$$

$$\boxed{B = 3}$$

$$y_p = -2t + 3$$

$$\text{So, } \boxed{y = -2t + 3 + c_1 e^t + c_2 e^{4t}}$$

IVP: $y(0) = -1 \quad y'(0) = -3$

$$-1 = -2(0) + 3 + c_1 e^0 + c_2 e^{4 \cdot 0} = 3 + c_1 + c_2$$

$$c_1 + c_2 = -4, \quad \boxed{c_1 = -4 - c_2}$$

$$\boxed{c_1 = -5}$$

$$y' = -2 + c_1 e^t + 4c_2 e^{4t} \Rightarrow -3 = -2 + c_1 + 4c_2$$

$$-1 = c_1 + 4c_2 \Rightarrow -1 = -4 - c_2 + 4c_2$$

$$\boxed{c_2 = 1}$$

So the particular sol'n is $y = -2t + 3 - 5e^t + e^{4t}$