Test #1 Solutions

Problem 3

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Find the general sol'n for:

$$t^2y' + 3ty = -8y^2$$

$$y' + \frac{3t}{t^2}y = -\frac{8}{t^2}y^2$$

$$y' + \underbrace{\frac{3}{t}}_{p(x)}y = \underbrace{\frac{-8}{t^2}}_{f(x)}y^{2} \leftarrow r=2$$

(This is a Bernoulli eqn)

① Solve assoc. homogeneous equation:

$$y_1' + \underbrace{\frac{3}{t}}_{p(x)}y_1 = 0$$

$$y_1 = ce^{-\int p(x)\,dx}$$

$$= ce^{-\int \frac{3}{t}\,dt}$$

$$= ce^{-3\int \frac{1}{t}\,dt}$$

$$= ce^{-3\ln|t|+k} \quad \text{(drop } |t| \text{ bc we are after } y_1\text{, a soln)}$$

$$= ct^{-3}$$

$$y_1 = \frac{c}{t^3}. \text{ Let } c=1. \quad \boxed{y_1 = \frac{1}{t^3}}$$

② Var. of params. $y = \underbrace{u}_{\text{mystery fn}} \cdot y_1$

$$\frac{u'}{u^r} = f(x)y^{r-1}$$

$$\frac{u'}{u^2} = -\frac{8}{t^2}y^{2-1}$$

$$\frac{1}{u^2}u' = -\frac{8}{t^2}\left(\frac{1}{t^3}\right)$$

$$\int \frac{1}{u^2}\,du = -8\int \frac{1}{t^5}\,dt$$

$$\int u^{-2}\,du = -8\int t^{-5}\,dt$$

$$-u^{-1} + c_1 = -8\left(-\frac{1}{4}t^{-4} + c_2\right)$$

$$-\frac{1}{u} + c_1 = 2t^{-4} + c_2$$

$$-\frac{1}{u} = 2t^{-4} + c$$

$$-1 = u(2t^{-4} + c)$$

$$u = \frac{-1}{2t^{-4} + c}$$

$$\boxed{y = u \cdot y_1 = \left(\frac{-1}{2t^{-4}+c}\right) \cdot \frac{1}{t^3}}$$