Phoebe Narcisse

$y = \sum_{n=0}^{\infty} a_n x^n \quad y' = \sum_{n=0}^{\infty} n a_n x^{n-1} \quad y'' = \sum_{n=0}^{\infty} n(n-1) a_n x^{n-2}$ } Will always be the same

· Power Series

Taylor: $T(x) = \sum_{n=0}^{\infty} \frac{y^{(n)}(a)}{n!}(x-a)^n$ Maclaurin Series is $T(x)$ when $x = 0$ $y(x) = T(x)$ in interval of convergence

Calculation: Let $y = \sum_{n=0}^{\infty} a_n x^n$ be series rep. of $y = y(x)$ on interval I containing 0

(1) Express ☆ as power series on interval I

ex. ☆ → $(2-x)y'' + 2y$

1x. first is $n=2$, second is $n=1$ — a. plug deriv. of y and plug into ☆, write as LS = ☆ (ex. $(2-x)\sum_{n=0}^{\infty} n(n-1)a_n x^{n-2} + 2\sum_{n=0}^{\infty} a_n x^n$)

b. distribute outside of sigma seperately (ex. $\sum 2n(n-1)a_n x^{n-2} - \sum n(n-1)a_n x^{n-1}$ + ^ ^ ^) ☆☆

n = same #) ↔ index — c. shift index to make all x terms = x^n (ex: $\sum 2(n+2)(n+1)a_{n+2}x^n - \sum (n+1)n a_{n+1} x^n$ + ^ ^ ^) ← shift

d. combine like terms (ex: $\sum_{n=0}^{\infty}[2(n+2)(n+1)a_{n+2} - (n+1)n a_{n+1} + 2a_n] x^n$

(2) Find coefficients for ☆

a. set inside brackets (1d) to equal 0 (ex: $2(n+2)(n+1)a_{n+2} - (n+1)n a_{n+1} + 2a_n = 0$)

* a_0 and a_1 are free var. — b. solve/isolate the highest a_n term (ex. $a_{n+2} = [(n+1)n a_{n+1} - 2a_n]/[2(n+2)(n+1)]$)

c. plug in $n=0, n=1$, etc to find a coefficients (ex. $n=0$: $a_2 = -\frac{1}{2}a_0$)

d. write out the series rep. of sol'n (ex. $y = a_0 + a_1 x + a_2 x^2 + a_3 x^3 + \ldots$) ← you will be plugging in a_2, a_3 ...

e. combine like terms (ex. $y = a_0(\ldots + \ldots) + a_1(\ldots + \ldots)$)

☆=0 → IVP — (3) Find series rep. of ☆ = 0

ex: $y = \sum_0^{\infty} a_n x^n = a_0 + a_1 x + a_2 x^2 + \ldots$, $y(0) = \sum_0^{\infty} a_n 0^n = a_0 = 1$

$y' = \sum_0^{\infty} n a_n x^{n-1} = 0 + a_1 + 2a_2 x + 3a_3 x^2 + \ldots$, $y'(0) = \sum_0^{\infty} n a_n 0^{n-1} = a_1 = -1$

~~~~ sub. $y(0)$ for a_0 and $y'(0)$ for a_1 in 2e, then combine like terms for final ans.

SPECIAL CASE: even indices: $n = 2m$ $(a_2, a_4, a_6 \ldots)$

$a_{n+2} = -a_n/(n+2)(n+1) \rightarrow a_0, a_1, a_2, a_3$ odd ^^ : $n = 2m+1$ $(a_3, a_5, a_7 \ldots)$

☆☆ Distribution

$(x+2)\sum_0^{\infty} n(n-1)a_n x^{n-2}$ distribute $(x+2)$ seperately, $2x\sum_0^{\infty} n(n-1)a_n x^{n-2}$ distribute $2x$ itself

Ordinary Point open interval $(x_0 - \rho, x_0 + \rho)$

Ex. 1 $(x+6)y'' - (7-x)y' + y = 0$

$P_0(x) = x+6 = 0$, $x = -6$ $\rho = |0-(-6)| = 6$ interval: $(0-6, 0+6) = (-6, 6)$

*** mag. of complex $a+bi = \sqrt{a^2+b^2}$

Ex. 2 $(x^2+1)y'' - (7-x)y' + y = 0$

$P_0(x) = x^2+1 = 0$, $x = \pm i$ $\rho = |0-i| = 1$, $\rho = |0+i|$ interval: $(0-1, 0+1) = (-1, 1)$ ***

S.C. (cont.)

↳ Find general m (ex. even: $a_{2m} = (-1)^m \frac{a_0}{(2m)!}$, odd: $a_{2m+1} = (-1)^m \frac{a_1}{(2m+1)!}$

follow rest of same steps in Power Series
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