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## 5.2 CONSTANT COEFFICIENT HOMOG. EQ'NS

$ay'' + by' + cy = 0$

char. poly

$ar^2 + br + c = 0 \leftarrow$ QUAD. EQ'N $\therefore r = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$

CASE 1: $r_1 \neq r_2$, $y_1 = e^{r_1 x}$ $y_2 = e^{r_2 x}$

CASE 2: $r_1 = r_2$, $y_1 = e^{r_1 x}$ $y_2 = xe^{r_1 x}$

CASE 3: $r_1 = \lambda + i\omega$, $r_2 = \lambda - i\omega$, $y_1 = e^{\lambda x}\sin(\omega x)$ $y_2 = e^{\lambda x}\cos(\omega x)$

example

Solve $y'' + 4y' + 13y = 0 \rightarrow$ char. poly: $r^2 + 4r + 13 = 0$ ... $r = \frac{-b \pm \sqrt{\ }}{2a} = -2+3i$ $\therefore r_1 = -2+3i$, $r_2 = -2-3i$

$\lambda = -2$ (real part) $\omega = 3$ (imaginary part) $\therefore y_1 = e^{-2x}\sin(3x)$ $y_2 = e^{-2x}\cos(3x) \rightarrow$ gen. sol'n: $c_1 e^{-2x}\sin(3x) + c_2 e^{-2x}\cos(3x)$

Solve gen. sol'n $y'' + 2y' + y = 0 \rightarrow$ c.p.: $r^2 + 2r + 1 = 0$ ... $(r+1)^2 = 0$ $\therefore r_1 = -1, r_2 = -1$

$y_1 = e^{-x}$ $y_2 = xe^{-x} \rightarrow$ gen. sol'n: $y = c_1 e^{-x} + c_2 x e^{-x}$

## 5.3 NON-HOMOG. LINEAR 2ND ORDER EQ'NS

$y'' + p(x)y' + q(x)y = f(x)$

Gen. Sol'n: $y = y_p + c_1 y_1 + c_2 y_2$

$y_p$ determined by $f(x)$, ex. if $f(x)$ = linear then $y_p = Ax + B$, if $f(x)$ = quad., $y_p = Ax^2 + Bx + C$

STEP 1: Find char. poly like in 5.2, determine the case, you should have $y_1$ & $y_2$

STEP 2: Find part. sol'n $y_p$, if quad: $y_p = Ax^2 + Bx + C$, $y_p' = 2Ax + B$, $y_p'' = 2A$

plug into ★ (LS of $f(x)$), simplify then make = RS, use matrix to add like coefficients, find A, B, C,

Plug back into $y_p$ $\therefore y = Ax^2 + Bx + C + c_1 y_1 + c_2 y_2 \leftarrow$ Gen. sol'n

if linear: $y_p = Ax + B$, $y_p' = A$, $y_p'' = 0$ follow same steps ↑

## 5.4 UNDETERMINED COEFFICIENTS

$ay'' + by' + cy = e^{\alpha x} G(x)$

$\alpha$: some # ← polynomial

DO STEP 1, then STEP 2: write $y = ue^{\alpha x}$, then find $y'$ and $y''$ from it ($y' = e^{\alpha x}(u' + \alpha u)$, $y'' = e^{\alpha x}(u'' + 2\alpha u' + 3\alpha u)$)

plug $y, y', y''$ into LS, STEP 3: follow 5.3 STEP 2 to find part. sol'n of $au'' + bu' + cu = G(x)$ [use $u_p$ instead of $y_p$]

STEP 4: multiply $u_p$ w/ $e^{\alpha x}$ so $(u_p)e^{\alpha x}$ ← this will be the part. soln final answer

## 5.6 REDUCTION OF ORDER

$y = u_p e^{\alpha x}$ ← gen. sol'n

$P_0(x)y'' + P_1(x)y' + P_2(x)y = 0$

Given one solution, $y_1$

STEP 1: write $y = uy_1$, $y' = u'y_1 + uy_1'$, $y'' = u''y_1 + u'y_1' + u'y_1' + uy_1'' \rightarrow$ plug these into LS

simplify then make $u' = z$, $u'' = z'$ for substitution

① DO STEP 1, DET. CASE → 5.7 Variation of Parameters

② DO V. OP.

$P_0(x)y'' + P_1(x)y' + P_2(x)y = f(x)$ ★

set $y_p = u_1 y_1 + u_2 y_2$ $y = y_p + c_1 y_1 + c_2 y_2 \leftarrow$ gen. sol'n

Assume ① $u_1' y_1 + u_2' y_2 = 0$, find $y_p'$ & $y_p''$ & plug into ★ to get eq ② LS=RS

Solve using matrix ①② , after getting $u_1'$ or $u_2'$, Integrate for $u_1$ or $u_2$, then find the other one

plug findings into gen. sol'n for final answer

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$\frac{d}{dx}(\ln|x|) = \frac{1}{x}$, $\int \frac{1}{x}dx = \ln|x|$ $\frac{d}{dx}(\sin x) = \cos x$, $\int \cos x\,dx = \sin x$ $\frac{d}{dx}(\cos x) = -\sin x$, $\int \sin x = -\cos x$

$\frac{d}{dx}\tan x = \sec^2 x$, $\int \sec^2 x\,dx = \tan x$ $\frac{d}{dx}\sec x = \sec x \tan x$, $\int \sec x \tan x\,dx = \sec x$

$\frac{d}{dx}(\sin^{-1}) x = \frac{1}{\sqrt{1-x^2}}$, $\int \frac{1}{\sqrt{1-x^2}}dx = \sin^{-1}x$ $\frac{d}{dx}\tan^{-1}x = \frac{1}{1+x^2}$, $\int \frac{1}{1+x^2}dx = \tan^{-1}x$

$\frac{d}{dx}(\sec^{-1}|x|) = \frac{1}{x\sqrt{x^2-1}}$, $\int \frac{1}{x\sqrt{x^2-1}}dx = \sec^{-1}|x|$

IBP $\int u\,dv = u \cdot v - \int v \cdot du$ ① use LIATE to find u, ② simplify then the rest is dv; use sub. if still complex

$\sin\theta = \frac{1}{\csc}$ $\cos\theta = \frac{1}{\sec}$ $\tan\theta = \frac{1}{\cot\theta}$ $\csc\theta = \frac{1}{\sin}$ $\sec = \frac{1}{\cos}$ $\tan = \frac{\sin}{\cos}$ $\cot = \frac{\cos}{\sin}$ $\frac{d}{d\theta}\sec\theta = \frac{\sin\theta}{\cos^2\theta}$