Phoebe Narcisse

- Linear 1st Order Diff. Eq'ns
  1. Standard Form: $y' + p(x)y = q(x)$
  2. Identify $p(x)$ & $q(x)$
  3. Determine the integrating factor: $I(x) = e^{\int p(x)dx}$
  4. Write the gen. sol'n

     $y = \frac{1}{I(x)}\left[\int I(x)q(x)dx + C\right]$

- Homogenous Linear 1st Order Eq'ns
  $\hookrightarrow$ when $y' + p(x)y = 0$
  then $y = ce^{-\int p(x)dx}$ ← Gen. Soln
- Seperable EQNS
  1. seperate variables (y on LS, x on RS)
  2. integrate both sides
  3. Solve for y
- LINEAR NON-HOMOGENOUS EQ'NS
  1. Find a sol'n of the associated homog. eq'n (a.k.a make it homog. then follow this rule
  2. Use variation of parameters to find u:

     $u' = \frac{q(x)}{y_1}$ ← y_1 comes from STEP 1!
  3. Write general soln as:

     $y = uy_1$
- Bernoulli EQ'NS

  Standard form: $y' + p(x)y = f(x) \cdot y^r$
  1. y_1 is any soln of $y' + p(x) = 0$ [see 1 in LINEAR NON-HOMOG. EQN'S]
  2. V.o.P: $\frac{u'}{u^r} = f(x)y_1^{r-1}$
  3. Write general soln as:

     $y = uy_1$
- NONLINEAR HOMOGENOUS EQ'NS
  1. Substitute $y = ux$, $y' = u'x + u$ into eq'n
  2. Solve for $u'x + u$, then simplify and seperate variables (u on LS, x on RS)
  3. V.o.P. $\frac{u'}{(u'x+u)-u} = \frac{1}{x}$, substituting $(u'x+u)$
  4. Integrate both sides and solve for u
  5. Final answer should look like: $y = ux$
- EXACT EQ'NS
  1. Identify $M(x,y)$ & $N(x,y)$
  2. For $M_y(x,y)$: integrate y's + simplify product (leave out sums)

     For $N_x(x,y)$: integrate x's + simplify product ^ ^ ^

     ↳ If they are the same, the eq'n is exact
  3. RECALL: Theorem implies $F_x(x,y) = M_y(x,y) + F_y(x,y) = N(x,y)$
  4. To find F: integrate x's (so integrate $M(x,y)$) + should have "$+\phi y$" not "$+C$"
  5. Rewrite $F_y(x,y) = \sim + \phi y$, integrate y's to find $\phi' y = \sim$ (leave out any x's)
  6. Differentiate $\phi' y$ to find ϕy (ex. $\phi' y = 6y^2$, $\phi = 2y^3$)
  7. Sub. ϕy into step 5 eq'n ($F_y(x,y) = \sim + \phi y$)
  8. Final ans. should look like: $C = F_y(x,y)$

Particular Solns?
1. Sub in x+y, solve for C, rewrite eq'n

IbP?

$u \cdot v - \int v \cdot du$

★ use LIATE to find u, the rest is dv

Integrals:

$\int \frac{1}{x}dx = \ln|x| + C$, $\int k\,dx = kx + C$, $\int \frac{1}{y^2+1} = \tan^{-1}(y) + C$

NOTE:

Ex → $e^{\int 4/x\,dx} = e^{4\int 1/x\,dx} = e^{4\ln|x|} = e^{\ln(x)^4} = x^4$ OR $e^{\int 1/x\,dx} = e^{\ln(x)} = x$

MORE INTEGRALS:

$\int \frac{1}{1+x}dx = \ln|1+x| + C$, $\int e^{ax}dx = \frac{1}{a}e^{ax} + C$