Test #4 Solutions

Problem 5

$y(0)=3$, $y'(0)=2$. Plug in to previous section.

Previously, we got

$$a_0\left(1-x^2+\frac{1}{3}x^3-\frac{1}{12}x^4+\frac{1}{12}x^5+\dots\right)+a_1\left(x-\frac{1}{2}x^2-\frac{1}{6}x^3+\frac{1}{24}x^4-\frac{1}{24}x^5+\dots\right)$$

Let's plug in our IVP to y and y'.

$$y=\sum_{n=0}^{\infty}a_nx^n$$

$$3=\sum_{n=0}^{\infty}a_n(0)^n=a_0(0)^0+\underbrace{a_1(0)^1+a_2(0)^2+\cdots}_{=0}$$

$$\boxed{a_0=3}$$

$$2=y'=\sum_{n=0}^{\infty}(n)a_n(0)^{n-1}=\cancel{(0)a_0(0)^{-1}}+(1)a_1(0)^0+\cancel{(2)a_2(0)^1}+\dots$$

$$\boxed{a_1=2}$$

So the series solution of $(x^2-1)y''-y'-2y=0$ with $y(0)=3$ and $y'(0)=2$ is:

$$3\left(1-x^2+\frac{1}{3}x^3-\frac{1}{12}x^4+\frac{1}{12}x^5+\dots\right)+2\left(x-\frac{1}{2}x^2-\frac{1}{6}x^3+\frac{1}{24}x^4-\frac{1}{24}x^5+\dots\right)$$

$$=3-3x^2+x^3-\frac{1}{4}x^4+\frac{1}{4}x^5+2x-x^2-\frac{1}{3}x^3+\frac{1}{12}x^4-\frac{1}{12}x^5$$

$$\boxed{=3+2x-4x^2+\tfrac{2}{3}x^3-\frac{1}{6}x^4+\frac{1}{6}x^5+\dots}$$