Section 6.3 The RLC Circuit **Wellington Verduga & Phoebe Narcisse**

Question 3: Find the current in the RLC circuit, assuming that $E(t) = 0$ for $t > 0$, given R = 2Ω L = 0.1H C = 0.01F Q_0 = 2C I_0 = 0A

This problem involves free oscillation as $E(t) = 0$ for $t > 0$ and thus we use equation 6.3.8 from our book:

$$LQ'' + RQ' + \frac{1}{C}Q = 0.$$

Plugging our given values we get end with equation:

$$0.1Q'' + 2Q' + \frac{1}{0.01}Q = 0.$$

First, we will look at the characteristic polynomial of our equation, which is:

$$0.1r^2 + 2r + \frac{1}{0.01} = 0,$$

which has complex roots given by the quadratic formula

$$r = \frac{-2 \pm \sqrt{2^2 - (4 * 0.1 * \frac{1}{0.01})}}{2 * 0.1}; \; r_1 = -10 + 30i \; and \; r_2 = -10 - 30i.$$

Then, we plug this into our solution for constant coefficients homogenous second order differential equations and we obtain:

$$Q = e^{-10t}(C_1 \cos(30t) + C_2 \sin(30t)).$$

Let's differentiate the equation so that we can then plug in our initial values and obtain our C_1 and C_2.

Using the Power rule to differentiate:

$$Q' = \frac{dQ}{dt}(e^{-10t}(C_1 \cos(30t) + C_2 sin(30t)))$$

$$Q' = \frac{dQ}{dt}(e^{-10t}) * (C_1 \cos(30t) + C_2 \sin(30t)) + e^{-10t} * \frac{dQ}{dt}(C_1 \cos(30t) + C_2 \sin(30t))$$

$$Q' = -10e^{-10t}(C_1 \cos(30t) + C_2 \sin(30t)) + e^{-10t}(30C_2 \cos(30t) - 30C_1 \sin(30t).$$

Our initial values are Q_0 = 2C and I_0 = -2A, solve for C_1 and C_2:

$$Q(0) = e^{-10(0)}(C_1\cos(30(0)) + C_2\sin(30(0))) = 2;$$

$$e^0(C_1\cos(0) + C_2\sin(0)) = 2\,;$$

$$1\big(C_1(1) + C_2(0)\big) = 2;\ C_1 = 2$$

Using our now found C_1 and I_0 = 2A:

$$Q'(0) = -10e^0(2\cos(0) + C_2\sin(0)) + e^0(30C_2\cos(0) - 30(2)\sin(0)) = 0$$

$$Q'(0) = -10(2+0) + (30C_2) = -20 + 30C_2 = 0;\ C_2 = \frac{20}{30} = \frac{2}{3}.$$

Plugging our C_1 and C_2 back into our general solution gives us:

$$Q = e^{-10t}(2\cos(30t) + \frac{2}{3}\sin(30t)),$$

which now we differentiate by power rule once again to obtain our current and our solution:

$$\frac{dQ}{dt} = \frac{dQ}{dt}(e^{-10t}(2\cos(30t) + \frac{2}{3}\sin(30t))$$

$$\frac{dQ}{dt} = (\frac{dQ}{dt}(e^{-10t}) * \left(2\cos(30t) + \frac{2}{3}\sin(30t)\right)) + (e^{-10t} * \frac{dQ}{dt}(\left(2\cos(30t) + \frac{2}{3}\sin(30t)\right))$$

$$Q' = -10e^{-10t}(2\cos(30t) + \frac{2}{3}\sin(30t)) + e^{-10t}(20\cos(30t) - 60\sin(30t)$$

$$Q' = -e^{-10t}(10\left(2\cos(30t) + \frac{2}{3}\sin(30t)\right) - 20\cos(30t) + 60\sin(30t))$$

$$Q' = -e^{-10t}(20\cos(30t) + \frac{20}{3}\sin(30t) - 20\cos(30t) + 60\sin(30t))$$

$$Q' = -e^{-10t}(\frac{180+20}{3}\sin(30t))$$

$$\boldsymbol{I = -\frac{200}{3}e^{-10t}sin(30t)}$$