Benjamin Tolbert – MAT 2680 Differential Equations

Project #2 – Section 6.3 – Problem #2 – 11/20/2023

$Find$ the current in the RLC circuit, assuming that assuming that $\mathrm{E(t)} = 0$ for $\mathrm{t} > 0$.
2) $R = 2\ ohms$; $L = .05\ henrys$; $C = .01\ farads$; $\mathrm{Q}_0 = 2\ coulombs$; $\mathrm{I}_0 = -2\ coulombs$

Since we know the values for R, L, and C, & $\mathrm{E(t)} = 0$ for $\mathrm{t} > 0$, that must mean the RLC circuit is in free oscillation. In that case, the values must be plugged into the equation:

$$\mathrm{LQ}'' + \mathrm{RQ}' + \frac{1}{\mathrm{C}}\mathrm{Q} = 0$$

$$0.5\mathrm{Q}'' + 2\mathrm{Q}' + 100\mathrm{Q} = 0$$

When converted, the characteristic equation is:

$$0.5\mathrm{r}^2 + 2\mathrm{r} + 100 = 0$$

Afterwards, the roots for the characteristic equation must be found:

$$r = \frac{-R \pm \sqrt{R^2 - 4L/C}}{2L}$$

$$r = \frac{-(2) \pm \sqrt{(2)^2 - 4(0.5)(100)}}{2(0.5)}$$

$$r = -2 \pm \sqrt{-196}$$

$$r = -2 \pm 14i$$

$$r_1 = -\frac{R}{2L} + i\omega_1;\ r_2 = -\frac{R}{2L} - i\omega_1$$

$$r_1 = -2 + 14i;\ r_2 = -2 - 14i$$

In the case of this circuit, it is underdamped as $\mathrm{R} < \sqrt{4L/C}$ $(2 < \sqrt{200})$. The roots, r1 and r2 have also been found. From there, the general solution is:

$$Q = e^{-Rt/2L}(c_1 cos\omega_1 t + c_2 sin\omega_1 t)$$

$$Q = e^{-2t}(c_1\cos(14t) + c_2\sin(14t))$$

Differentiating the solution and collecting like terms gives:

$$Q' = -2e^{-2t}\big(c_1\cos(14t) + c_2\sin(14t)\big) + 14e^{-2t}(-c_1\sin(14t) + c_2\cos(14t))$$

To solve the initial value problem of $Q_0 = 2\ coulombs; I_0 = -2\ coulombs$, t must be set to 0 for Q and Q' to equal $Q_0\ and\ I_0$ respectively.

$$Q(O) = e^{-2(0)}\left(c_1\cos\left(14(0)\right) + c_2\sin\left(14(0)\right)\right) = 2$$

$$Q(O) = e^0\left(c_1\cos(0) + c_2\sin(0)\right) = 2$$

$$Q(O) = 1\left(c_1\cos(0) + c_2\sin(0)\right) = 2$$

$$Q(O) = 1(c_1 + 0) = 2$$

$$Q(O) = c_1 = 2$$

$$c_1 = 2$$

With c_1 found, it can be plugged into Q' to find c_2.

$$Q'(0) = -2e^{-2(0)}\left(c_1\cos\left(14(0)\right) + c_2\sin\left(14(0)\right)\right) + 14e^{-2(0)}\left(-c_1\sin\left(14(0)\right) + c_2\cos\left(14(0)\right)\right) = -2$$

$$Q'(0) = -2e^0\left(c_1\cos(0) + c_2\sin(0)\right) + 14e^0\left(-c_1\sin(0) + c_2\cos(0)\right) = -2$$

$$Q'(0) = -2(c_1 + 0) + 14(0 + c_2) = -2$$

$$Q'(0) = -2(c_1 + 0) + 14(0 + c_2) = -2$$

$$Q'(0) = -2c_1 + 14c_2 = -2$$

$$Q'(0) = -2(2) + 14c_2 = -2$$

$$Q'(0) = -4 + 14c_2 = -2$$

$$Q'(0) = 14c_2 = 2$$

$$Q'(0) = c_2 = \frac{2}{14}$$

$$c_2 = \frac{1}{7}$$

With both c_1 and c_2 found, it can be plugged into the solution, Q to give:

$$Q = e^{-2t}(2\cos(14t) + \frac{1}{7}\sin(14t))$$

Finally, differentiating Q will yield the current, I.

$$Q' = -2e^{-2t}\left(2cos(14t) + \frac{1}{7}\ sin(14t)\right) + 14e^{-2t}\left(-2\sin(14t) + \frac{1}{7}e^{-2t}\cos(14t)\right)$$

$$Q' = -4e^{-2t}\cos(14t) - \frac{2}{7}e^{-2t}\sin(14t) - 28e^{-2t}\sin(14t) + 2e^{-2t}\cos(14t)$$

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$$Q' = -2e^{-2t}\cos(14t) - \frac{198}{7}e^{-2t}\sin(14t)$$

$$I = e^{-2t}\left(-2\cos(14t) - \frac{198}{7}\sin(14t)\right)$$