

6.1: Springs I

Equation of Motion:

$$my'' + cy' + ky = F$$

mass damping force spring constant

Question 6.1 An object stretches a spring 4 inches in equilibrium. Find and graph its displacement for $t > 0$ if it's initially displaced 36 inches above equilibrium and given a downward velocity of 2 ft/s.

Assume: no damping $\Rightarrow c = 0$

$$y(0) = 3 \text{ ft} \quad \text{and} \quad y'(0) = -2 \text{ ft/s}$$

$$my'' + ky = 0$$
$$y'' + \frac{k}{m}y = 0$$

$$y'' + 96y = 0$$
$$r^2 + 96 = 0$$
$$r = \pm \sqrt{96}i$$

$$mg = k\Delta l$$

$$\frac{k}{m} = \frac{g}{\Delta l}$$

$$\frac{k}{m} = \frac{32 \text{ ft/s}^2 \cdot 96}{1/3 \text{ ft}} = 96$$

$$y = C_1 \sin(4\sqrt{6}t) + C_2 \cos(4\sqrt{6}t)$$

$$y' = 4\sqrt{6}C_1 \cos(4\sqrt{6}t) - 4\sqrt{6}C_2 \sin(4\sqrt{6}t)$$

$$y(0) = C_2 = 3$$

$$y'(0) = 4\sqrt{6}C_1 = -2$$
$$C_1 = -\frac{1}{2\sqrt{6}}$$

$$\Rightarrow y = -\frac{1}{2\sqrt{6}} \sin(4\sqrt{6}t) + 3 \cos(4\sqrt{6}t)$$