MAT 2680 Differential Equations Project 1

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October 2023

1 Section 4.1 Problem 13

A super bread dough increases in volume at a rate proportional to the volume V present. If V increases by a factor of 10 in 2 hours and $V(0) = V_0$, find V at any time t. How long will it take for V to increase to $100V_0$?

We know V is increasing by a constant rate, so the rate of change in V, or V', will be equivalent to the volume present V proportional to a constant, which we'll call k.

Writing that out, we have $V' = kV$.

$V' = kV$ can be rewritten into a homogenous linear equation, $V' - kV = 0$. Accordingly, we know that $p(t) = -k$, and can solve for our general solution:

$$\begin{aligned} V &= ce^{-\int p(t)dt} \\ &= ce^{-\int -kdt} \\ &= ce^{\int kdt} \\ &= ce^{kt}. \end{aligned}$$

Our general solution is $\boxed{V = ce^{kt}}$.

We're given some initial values that we can use to find a particular solution. We learn that V increases by a factor of 10 in two hours, and we learn that $V(0) = V_0$. Let's first take a look at $V(0) = V_0$.

We can find $V(0)$ by plugging in $t = 0$ to $V = ce^{kt}$ on the right side of our general solution:

$$ce^{k(0)} = V_0$$
$$e^{k(0)} = 1$$
$$c \cdot 1 = V_0$$

So that leaves us with the expression: $c = V_0$.

Since these two are equal, we can then plug in V_0 for c in our general solution and come out with $V = V_0e^{kt}$.

We still have other initial values to work with, so let's use those to find our particular solution.

We're given that V increases by a factor of 10 in 2 hours. Another way to think about this is that the initial volume V_0 is multiplied by 10 to reach the final volume V after 2 hours: $V(2) = 10V_0$.
Let's plug this into our previous function $V = V_0 e^{kt}$ and solve for k.

$$
\begin{aligned}
V &= V_0 e^{kt} \\
10V_0 &= V_0 e^{2k} \\
10 &= e^{2k} \\
\ln(10) &= 2k \\
k &= \frac{1}{2}\ln(10).
\end{aligned}
$$

This is our particular solution - the equation to find V at any time t:

$$\boxed{V = V_0 \cdot e^{\frac{1}{2}\ln(10)t}}.$$

We now have a particular solution that can find V at any time t. But how long will it take for V to increase to $100V_0$?

Like last time, we can represent this increase as $V = 100V_0$. Using our new particular solution that we just got, let's set up our equation to say:

$$
\begin{aligned}
V_0 e^{\frac{1}{2}ln(10)t} &= 100 \cdot V_0 \\
e^{\frac{1}{2}ln(10)t} &= 100 \\
\frac{1}{2}ln(10)t &= ln(100) \longrightarrow ln(100) = 2ln(10) \\
\frac{1}{2}ln(10)t &= 2ln(10) \\
\frac{1}{2}t &= 2 \\
t &= 4
\end{aligned}
$$

Therefore, $\boxed{t = 4 \text{ hours}}$, so it will take 4 hours for the bread dough to increase to 100 times its initial volume.