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MAT 2680 Prof. POIRIER

Differential Equations in Science

Differential Equations or one of the most important subjects to know when you study science. Why is it important though? Lets first define what a differential equation is. Any differential equation is an equation with a function and one or more of its derivatives. Why use differentials at all? Differential equations have the fascinating ability to predict the world around us. This is very important especially in math and science fields. It can describe how populations change, how heat moves, how springs vibrate, how radioactive material decays and much more. Taking radioactive material decay and trying to find the mass is important. We can use a differential equation to solve for the mass of it. Where the substance is created at a rate of 25 g/hr, the substance decays at a rate of proportional to its mass with a proportionality of 0.875. Assuming it starts at 0g, we can write a function of time to determine the mass. As seen in the photograph on the next page, we can write a first order linear ODE to illustrate our situation. To find the mass we need to solve for Q(t). By deriving step by step our result gives up the equation we need. For this particular problem its (200/7) – 200/7e^ (7/8)t. With this equation which we found by deriving our initial data, we can find the mass substance just by inputting time. Its just that easy. Using differential equations, we solved for an equation that will give us the exact data we need for any time input. This is important in this line of work where time and ease of access of data is needed. This was only just one way to show a differentials importance in real life applications. Another example of importance is when we try to find a particle position. We are interested in the position x of this particle. Force on this particle will be g (gravity) minus ax' (drag is proportional to the speed), so F = -g + ax'. From knowing Newton's 2nd law that F = ma

= mx" (mass multiplied by the 2nd derivative of x wrt time). We have mx" = ax' - g. A 2nd order ODE whose solution solves the real-world problem of a particle falling under gravity and experiencing drag. Differentials are essential to science and is very recommended to take a class or two.

(10) 25 g/hr $\quad$ 0.875 $\quad Q(0)=0$

rate ↓

$Q(t)$ = mass at t $\quad$ Finding change of $Q(t)$ $\quad \frac{d}{dt}Q(t) = 25 - 0.875Q(t)$

rewrite ↓

$Q'(t) + 0.875\,Q_t = 25$ $\quad p = 0.875$

First order ODE

multiply equation by IF $\quad IF = e^{\int 0.875\,dt}$

$= e^{0.875t}$

$Q'(t)\,e^{0.875t} + 0.875Q(t)\,e^{0.875t} = 25e^{0.875t}$

Product rule

$(e^{0.875t}Q)' = 25e^{0.875t}$

$e^{0.875t}Q = \int 25e^{0.875t}\,dt$

rewrite $0.875 = \frac{7}{8}$

↓

$e^{\frac{7}{8}t}Q = \int 25e^{\frac{7}{8}t}\,dt$

↓

$25\int e^{\frac{7}{8}t}\,dt$

Substitution $u = \frac{7}{8}t$

$25\int \frac{8}{7}e^{u}\,du$

$25(\frac{8}{7})\int e^{u}\,du$

$25(\frac{8}{7})\,e^{u}$ $\quad$ Sub back

$25\cdot\frac{8}{7}e^{\frac{7}{8}t} = \frac{200}{7}e^{\frac{7}{8}t} + C$

0.875

↓

$e^{\frac{7}{8}}Q = \frac{200}{7}e^{\frac{7}{8}t} + C$

Isolate Q

$Q = \frac{200}{7} + \frac{C}{e^{7/8t}}$

Initial condition

$Q(0) = 0$

$0 = \frac{200}{7} + \frac{C}{e^{\frac{7}{8}(0)}}$ $\quad e^{0} = 1 \quad \frac{200}{7} + C = 0$

$C = \frac{-200}{7}$

Final

$Q = \frac{200}{7} - \frac{200}{7e^{\frac{7}{8}t}}$