Differential Equations of Insurgency Operations

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Mathematics in general has a vast array of tools that can be used to accomplish the most diminutive and insignificant problems. As well it can be used on the more mysterious and esoteric problems that are involved in string theory. Now Differential Equations can be very helpful in solving problems that involve economics, physics, engineering, and many other fields. For example as you will find in the future differential equations can be used to solve physics problems involving spring and electric circuits. Differential Equations can also be used in problems involving Population growth and decay. This short essay will be focusing on a more specific and extrange use of Differential Equations that involves military warfare. This essay will give a few examples and go over Operations Research and how this is involved in the planning and executions of plans that lead to a specific goal.

First a look will be given and Fredric William Lanchester. Lanchester was born in England London on october 23, 1868, and eventually died in march 8, 1946. He was an engineer who pioneered in the creation of the english automobile as well as in the area of aeronautics and as well contributed to the development of operations research(Frederick). Eventually wrote "Aircraft in Warfare:The Dawn of the Fourth Arm" a series of articles published in the "Engineer" periodical from September and December 1914. These periodicals were the first examples of Operations Research, which are models of conflict where there is a use of mathematical formulas as tools for strategy in physical conflict(Kress). In more modern times the idea of Operations Research can be defined as the application of scientific methods to the management of various administrations(Operations Research). In general Operations Research is an area of expertise in any given company, military entity, or governmental branch that needs to make decisions in order to accomplish goals and objectives. Any given objectives and goals are accomplished by the use of scientific methods like the one that will be discussed shortly. The one and specific example that will be given has to do with the use of a pair of first order differential equations that are used to plan and take action around the interaction of a governmental military with insurgent military forces. For a starting example situated in the conflict of the Vietnam War this equation can be obtained in Marvin Schaffer memorandum title "Lanchester Models of Guerrilla Engagements" the formula is presented is:

$$\frac{dm}{dt} = -S_n(m,n)$$
$$\frac{dn}{dt} = -S_m(m,n)$$

Where S = killing power of ground forces engaged in battle, t = time, m = side m (for ambush, ambushed; for siege, defender), n = side n (for ambush, ambusher; for siege, attacker) (Schaffer,5). Where all variables are positive except for S because it represents reduction in the ranks of military forces.The paper goes further over the use of more complex equations with the purpose as it is stated in the paper to " Although the models and theory are not adequate to predict the outcome of an insurgency, they should prove useful for examining the credibility of casualty claims associated with such conflicts"(Schaffer, iii). The paper goes onto give further examples improving the equation to take into account other factors during what the paper calls Phase 2 Insurgency. In this example the Equation form before becomes:

$$(\frac{dm}{dt})_c = - k_n(t,m)n \ - \Sigma^i E_i(t,m)W_i(t)$$
$$(\frac{dm}{dt})_c = - k_m(t,n)m \ - \Sigma^i E_j(t,n)W_j(t)$$

Where k = direct fire weapon efficiencies, $E_{i,j}$ = supporting weapons efficiencies, $W_{i,j}$ supporting weapons strengths(Schaffer, 5).

Another example of the use of Lanchester's equation is found in Kress's paper title Analytic Modeling of Insurgencies it is stated "The first to capture the asymmetry feature in a Lanchesterian setting was Deitchman in his Guerrilla Warfare model, which is a pair of differential equations"(Kress, 5). The Differential Equation looks as follows:

$$G'(t) \ = \ -aI(t)$$
$$I'(t) \ = \ -\gamma G(t)\frac{I(t)}{P}$$

Where G(t) = (gouvernemental force) and I(t) = (insurgents), are the sizes of the conflicting military forces at time t respectively. P is the size of the civilian population. a, γ are attrition coefficients(actions that reduce the opposite forces numbers, support, or fighting power) and the signature of the insurgents. The proportion of the insurgent population is written as $\frac{I(t)}{P}$ (Kress, 5). In general what the equation proposes is the idea that government forces(G) need to reduce the size of the insurgents(I) at time (t) in order to have the upper hand in combat. Secondly the larger the population in which the insurgent force is spread amongst, the harder it is for the governmental forces to clearly target the insurgent forces.

As seen in the examples above Lancherster's equation can be improved and modified depending on the objective designated. Other examples of this can be given. In the paper titled "A Dynamic Model of Insurgency: The Case of the War in Iraq" that uses differential equations to describe the possible outcome of the War. Also in the paper title " Bits or Shots in Combat? The Generalized Deitchman Model of Guerrilla Warfare" looks to improve the equation by adding several other factors.

Overall the essay has looked over the use of differential equations on Operation Researches. Several examples have been given and mention which used Lanchester's equation as a basis of further development. As explained before the use of differential equations in Operation Research are able to help and accomplish goals, it may not be able to predict or the result of an armed conflict. However it may help to have into account other factors that may be of major importance towards the accomplishment of a final goal.

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