Practice Circuits Problem 2

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(1 point) A series circuit has a capacitor of $\frac{1}{20} \times 10^{-6}$ F and an inductor of 5 H. If the initial charge on the capacitor is 3×10^{-3} C and there is no initial current, find the charge on the capacitor $Q(t)$.

What is the differential equation for this closed circuit?

What is the formula for the charge on the capacitor?

$Q(t) =$ [help (formulas)]

$C = \frac{1}{20} \cdot 10^{-6}$ F

$L = 5$ H

$Q(0) = 3 \times 10^{-3}$ C

$R = 0\Omega \qquad E(t) = 0$

$LQ'' + RQ' + \frac{1}{C}Q = 0$

$5Q'' + \frac{1}{\frac{1}{20} \cdot 10^{-6}} Q = 0$

$5Q'' + 20{,}000{,}000\,Q = 0$

The Differential equation for the closed circuit

$5Q'' + 20000000\,Q = 0$

$5r^2 + 20000000 = 0$

$r^2 = -4000000$

$r = 0 \pm 2000i$

$\lambda = 0 \qquad \omega = 2000$

$Q(t) = e^{\lambda t}(C_1 \sin(\omega t) + C_2 \cos(\omega t))$

$Q(t) = e^{0t}(C_1 \sin(2000t) + C_2 \cos(2000t))$

$Q(t) = (C_1 \sin(2000t) + C_2 \cos(2000t))$

$Q'(t) = C_1 \cos(2000t) \cdot 2000 - 2000\,C_2 \sin(2000t)$

$Q(0) = 3 \times 10^{-3}$ C

$C_1 \sin(2000t) + C_2 \cos(2000t) = 3 \times 10^{-3}$

$t \Rightarrow 0$

$C_1 \sin(2000(0)) + C_2 \cos(2000(0)) = 3 \times 10^{-3}$

$C_1 \sin(0) + C_2 \cos(0) = 3 \times 10^{-3}$

$\boxed{C_2 = 3 \times 10^{-3}}$

$Q'(0) = 0$

$0 = C_1 \cos(2000t) \cdot 2000 - 2000\,C_2 \sin(2000t)$

$t \Rightarrow 0$

$C_1 \cos(2000t) \cdot 2000 - 2000\,C_2 \sin(2000t) = 0$

$C_1 \cos(2000(0)) \cdot 2000 - 2000\,C_2 \sin(2000(0)) = 0$

(1) (0)

$C_1(1) \cdot 2000 - 0 = 0$

$2000\,C_1 = 0$

$\boxed{C_1 = 0}$

Put everything together

$Q(t) = C_1 \sin(2000t) + C_2 \cos(2000t)$

$C_1 = 0$ and $C_2 = 3 \times 10^{-3}$

$Q(t) = (0)\sin(2000t) + (3 \times 10^{-3})\cos(2000t)$ (0)

$\boxed{Q(t) = 0.003 \cos(2000t)}$

Formula for the charge on the Capacitor