COMPUTATIONAL FLUID DYNAMICS

Computational fluid dynamics use data structures to solve issues of fluid flow like velocity, density, and chemical compositions. Gas and liquid flow behavior is quantified by partial differential equations representing conservation laws for mass, momentum and energy.

Everyday you can find fluid flows problems in meteorology, aerodynamic design, engine combustion and industrial processes. By using the applications of fluid dynamics, you can calculate the forces on the aircraft. Without computational fluid dynamincs, it will be hard for designers to improve in aerodyamic characteristics.

The equation you use to solve the problems in fluid dynamics is Navier Stokes equation. The equations below can be simplified by viscous actions to yield the Euler Equation.

Coordinates: (x,y,z)
Velocity Components: (u,v,w)
Time : t
Density: ρ
Total Energy: Et
Pressure: p
Stress: τ
Heat Flux: q
Reynolds Number: Re
Prandtl Number: Pr

Continuity:

$$\frac{\partial \rho}{\partial t} + \frac{\partial(\rho u)}{\partial x} + \frac{\partial(\rho v)}{\partial y} + \frac{\partial(\rho w)}{\partial z} = 0$$

X – Momentum:

$$\frac{\partial(\rho u)}{\partial t} + \frac{\partial(\rho u^2)}{\partial x} + \frac{\partial(\rho uv)}{\partial y} + \frac{\partial(\rho uw)}{\partial z} = -\frac{\partial p}{\partial x} + \frac{1}{Re_r}\left[\frac{\partial \tau_{xx}}{\partial x} + \frac{\partial \tau_{xy}}{\partial y} + \frac{\partial \tau_{xz}}{\partial z}\right]$$

Y – Momentum:

$$\frac{\partial(\rho v)}{\partial t} + \frac{\partial(\rho uv)}{\partial x} + \frac{\partial(\rho v^2)}{\partial y} + \frac{\partial(\rho vw)}{\partial z} = -\frac{\partial p}{\partial y} + \frac{1}{Re_r}\left[\frac{\partial \tau_{xy}}{\partial x} + \frac{\partial \tau_{yy}}{\partial y} + \frac{\partial \tau_{yz}}{\partial z}\right]$$

Z – Momentum

$$\frac{\partial(\rho w)}{\partial t} + \frac{\partial(\rho uw)}{\partial x} + \frac{\partial(\rho vw)}{\partial y} + \frac{\partial(\rho w^2)}{\partial z} = -\frac{\partial p}{\partial z} + \frac{1}{Re_r}\left[\frac{\partial \tau_{xz}}{\partial x} + \frac{\partial \tau_{yz}}{\partial y} + \frac{\partial \tau_{zz}}{\partial z}\right]$$

Energy:

$$\frac{\partial(E_T)}{\partial t} + \frac{\partial(uE_T)}{\partial x} + \frac{\partial(vE_T)}{\partial y} + \frac{\partial(wE_T)}{\partial z} = -\frac{\partial(up)}{\partial x} - \frac{\partial(vp)}{\partial y} - \frac{\partial(wp)}{\partial z} - \frac{1}{Re_r Pr_r}\left[\frac{\partial q_x}{\partial x} + \frac{\partial q_y}{\partial y} + \frac{\partial q_z}{\partial z}\right]$$
$$+ \frac{1}{Re_r}\left[\frac{\partial}{\partial x}(u\tau_{xx} + v\tau_{xy} + w\tau_{xz}) + \frac{\partial}{\partial y}(u\tau_{xy} + v\tau_{yy} + w\tau_{yz}) + \frac{\partial}{\partial z}(u\tau_{xz} + v\tau_{yz} + w\tau_{zz})\right]$$

$$\frac{\partial \rho}{\partial t} + \frac{\partial(\rho u)}{\partial x} +$$

The symbol d represent partial derivatives.

Background of CFD

The methods were first developed to solve the linearized potential equations. By using the 2D methods, airfoil were developed from using conformal transformations of the flow about a cylinder. One of the earliest type of resembling modern CFD are by Lewis Fry Richardson. The calculations used finite differences and divided the physical space in cells but the equations doesn't work. The developed of three dimensional methods is available when there is a computer. Furthermore, the first work using computers to model fluid flow is from Navier Stoke equations.

Reference

- https://www.grc.nasa.gov/www/k-12/airplane/nseqs.html
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