Spring Problem

13. $mg = 2$ lb

mass of spring given by: $m = \frac{W}{g} = \frac{2}{32} = \frac{1}{16}$

From Hooke's Law:

spring constant, $k = \frac{2}{1/2}$ $\quad (6 \text{ in} = \frac{6}{12} \text{ ft})$

$= 4 \text{ lb/ft}$

$\frac{1}{16}u'' + 4u = \sin(8t)$ $\quad$ initial condition: $u(0) = 2 \text{ in} = \frac{2}{12} = \frac{1}{6} \text{ ft}$

$u''(0) = 0$

$u'' + 64u = 16\sin(8t)$

$u''(r^2 + 64) = 0$

$r^2 + 64 = 0$

$(r+8)(r-8) = 0$

$r = \pm 8i$

$y_c = c_1 \cos 8t + c_2 \sin 8t$

$y_p = t(A\cos 8t + B\sin 8t)$

$y_p' = (A\cos 8t + B\sin 8t) - t(-8A\sin 8t + 8B\cos 8t)$

$y_p'' = (-8A\sin 8t + 8B\cos 8t) + (-8A\sin 8t + 8B\cos 8t) + t(-64A\cos 8t - 64B\sin 8t)$

$= -16(A\sin 8t - B\cos 8t) - 64t(A\cos 8t + B\sin 8t)$

$y_p'' + 64y_p = -16(A\sin 8t - B\cos 8t) - 64t(A\cos 8t + B\sin 8t) + 64(t(A\cos 8t + B\sin 8t))$

$-16(A\sin 8t - B\cos 8t) = 16\sin(8t)$

$-16A = 16$ (which implies $A = -1$)

$16B = 0$ (which implies $B = 0$)

so $y_p = t(-1\cos 8t + 0\sin 8t)$

$= -t\cos 8t$

Therefore solution is $u(t) = c_1\cos 8t + c_2\sin 8t - t\cos 8t$