

Webwork:

Basic second order

Nº 5

$$a) y'' - 8y' - 9y = 0 \quad y(0) = -14 \\ y'(0) = -46$$

Find the general and particular solution:

$$y'' - 8y' - 9y = 0$$

$$\text{Let } y = e^{rt} \quad y'' = r^2 e^{rt} \\ y' = r e^{rt}$$

So we have:

$$r^2 e^{rt} - 8(r e^{rt}) - 9e^{rt} = 0$$

$$e^{rt}(r^2 - 8r - 9) = 0$$

$$r^2 - 8r - 9 = 0$$

$$(r-9)(r+1) = 0$$

$$r = 9 \quad r = -1$$

$$\text{so } \underline{y(t) = A e^{9t} + B e^{-t}}$$

General solution

$$b) y(t)'' = 9A e^{9t} - B e^{-t}$$

$$\text{For } y(0) = -14$$

$$-14 = A e^{9 \cdot 0} + B e^{-0}$$

$$-14 = A + B$$

$$A = -B - 14$$

$$y'(t) = 9(-B-14)e^{9t} - B e^{-t}$$

$$-46 = 9(-B-14)e^{9 \cdot 0} - B e^{-0}$$

$$-46 = -9B - 126 - B$$

$$-46 + 126 = -10B$$

$$80 = -10B$$

$$B = -8$$

$$A = -(-8) - 14$$

$$A = 8 - 14$$

$$A = -6$$

Thus the particular solution is: $y = -6e^{9t} - 8e^{-t}$