Question Section 6.3, Question 2

Find the current in the RLC Circuit, assuming that $E(t)=0$ for $t>0$

$R = 2$ ohms; $L = 0.05$ henrys; $C = 0.01$ Farads; $Q_0 = 2$ Coloumbs, $I_0 = -2A$

Kirchoff's Law states that the sum of the voltage drops in a closed RLC circuit equals the impressed voltage.

$$LI' + RI + \frac{1}{C}Q = E(t)$$

This can be farther translated into $LQ'' + RQ' + \frac{1}{C}Q = E(t)$, since it is implied that $I = Q'$

- Firstly, $E(t)=0$; $E(t) = E_0 \cos wt$

$$LQ'' + RQ' + \frac{1}{C}Q = E_0 \cos wt$$

$$\therefore LQ'' + RQ' + \frac{1}{C}Q = 0$$

plug in values given

$$0.05Q'' + 2Q' + \frac{1}{0.01}Q = 0$$

$$0.05Q'' + 2Q' + 100Q = 0$$

↓

Characteric equation

$0.05D^2 + 2D + 100 = 0$ (divide by 0.05)

$D^2 + 40D + 2000 = 0$

$D = -20 + 40i$

General Solution = $Q(t) = e^{-20t}[C_1 \cos(40t) + C_2 \sin(40t)]$

So $Q(t) = e^{-20t}[C_1 \cos(40t) + C_2 \sin(40t)]$

$t = 0 \quad Q(0) = 2$

$Q(0) = e^0 [C_1 \cos(0) + C_2 \sin(0) = 2$

$\therefore \quad C_1 = 2$

$I(t) = Q'(t) = -20e^{-20t}[C_1 \cos(40t) + C_2 \sin(40t) +$

$e^{-20t}[-40 C_1 \sin(40t) \sout{C_2 \cos} + 40t\, C_2 \cos(40t)]$

We know $I_0 = -2$ Amperes, so set equations equal to each other; then find $Q'(0)$

$-2 = Q'(0) = -20e^0 [C_1 \cos(0) + C_2 \sin(0) +$

$e^0 [-40 C_1 \sin 0 + 40 \sout{\sin} C_2 \cos(0)]$

$-2 = -20[2) + 40C_2$

$-2 + 40 = 40C_2$

$C_2 = \frac{38}{40} = \frac{19}{20}$

$\therefore \quad Q(t) = e^{-20t}[2\cos(40t) + \frac{19}{20}\sin(40t)$