

Muharrazat
RLC

Project # 3

Problem #9

$$\frac{1}{10} Q'' + 6 Q' + 250 Q = 10 \cos 100t + 30 \sin 100t$$

$$\Rightarrow Q'' + 60 Q' + 2500 Q = 100 \cos 100t + 300 \sin 100t$$

(i) To find complimentary function

$$(D^2 + 60D + 2500) Q = 100 \cos 100t + 300 \sin 100t$$

The auxiliary equation

$$D^2 + 60D + 2500 = 0$$

$$D = \frac{-60 \pm \sqrt{3600 - 10000}}{2}$$

$$D = \frac{-60 \pm \sqrt{-6400}}{2} = \frac{-60 \pm 80i}{2}$$

$$D = -30 \pm 40i$$

The complimentary solution, $Q = e^{-30t} (c_1 \cos 40t + c_2 \sin 40t)$

(ii) To find particular solution

$$Q_p = \frac{100 \cos 100t + 300 \sin 100t}{(D^2 + 60D + 2500)}$$

$$\Rightarrow Q_p = \frac{100 \cos 100t}{D^2 + 60D + 2500} + \frac{300 \sin 100t}{D^2 + 60D + 2500}$$

$$\Rightarrow Q_p = \frac{100 \cos 100t}{(60D - 7500)} + \frac{300 \sin 100t}{(60D - 7500)} \quad [\because D^2 = 100^2]$$

$$\Rightarrow Q_p = \frac{100}{60} \frac{\cos 100t}{(D - 125)} + \frac{300}{60} \frac{\sin 100t}{(D - 125)}$$

$$\Rightarrow Q_p = \frac{5}{3} \frac{(D+125)}{D^2-125} \cos 100t + 5 \frac{(D+125)}{D^2+125} \sin 100t$$

$$\Rightarrow Q_p = \frac{5}{3} \frac{(D+125)}{-25625} \cos(100t) + \frac{5(D+125)}{-25625} \sin 100t \quad [\text{Again, } D^2 = -100]$$

$$\Rightarrow Q_p = \frac{1(-100 \sin 100t)}{-15375} + \frac{125 \cos 100t}{(-15375)} - \frac{(100 \cos 100t)}{5125} - \frac{125 \sin 100t}{5125}$$

$$\Rightarrow Q_p = \frac{100}{15375} \sin 100t - \frac{125 \cos 100t}{15375} - \frac{100 \cos 100t}{5125} - \frac{125 \sin 100t}{5125}$$

$$\Rightarrow Q_p = \frac{100 \sin 100t - 375 \sin 100t}{15375} - \frac{(125 \cos 100t + 300 \cos 100t)}{15375}$$

$$\Rightarrow Q_p = \frac{-275 \sin 100t}{15375} - \frac{425 \cos 100t}{15375}$$

General solution: Complementary function + Particular solution

$$Q = e^{-30} (c_1 \cos 40t + c_2 \sin 40t) - \frac{275 \sin 100t}{15375} - \frac{425 \cos 100t}{15375}$$

$$\text{Current, } I = \frac{dQ}{dt}$$

$$\Rightarrow I = e^{-30} (-40c_1 \sin 40t + 40c_2 \cos 40t) + (-30)(c_1 \cos 40t + c_2 \sin 40t) e^{-30} - \frac{275 \times 100}{15375} \cos 100t + \frac{425 \times 100}{15375} \sin 100t$$

$$\Rightarrow I = e^{-30} (40c_1 \sin 40t + 40c_2 \cos 40t - 30c_1 \cos 40t - 30c_2 \sin 40t) - \frac{27500}{15375} \cos 100t + \frac{42500}{15375} \sin 100t \quad (A)$$