Test #3 Review

practice Circuits.

webwork problem #2

A series circuit has a capacitor of $\frac{1}{27} \times 10^{-6}$ F and an inductor of 3H

If the initial charge on the capacitor is 1×10^{-2} and there is no initial current, Find the charge on the capacitor Q(t).

$C = \frac{1}{27} \times 10^{-6}$ F $\quad L = 3H \quad Q(0) = 1 \times 10^{-2}$ C

$Q'(0) = I(0) = 0A \quad R = 0\,\Omega \quad E(t) = 0$

$$LQ'' + RQ' + \frac{1}{C}Q = 0$$

$$3Q'' + 0Q' + \frac{1}{\left(\frac{1}{27} \times 10^{-6}\right)}Q = 0$$

$$\boxed{27000000\,Q + 3Q'' = 0}$$ ← differential equation for closed circuit

$$3Q'' + 27000000\,Q = 0$$
$$3r^2 + 27000000 = 0$$
$$\frac{3r^2}{3} = \frac{-27000000}{3}$$

$$3Q'' + 27000000\,Q = 0$$
$$3r^2 + 27000000 = 0$$
$$\frac{3r^2}{3} = \frac{-27000000}{3}$$
$$r^2 = 9000000$$
$$\sqrt{r^2} = \sqrt{9000000} \qquad \lambda \quad w$$
$$r = \pm 3000i \quad \text{or} \quad 0 \pm 3000i$$

$$Q(t) = e^{\lambda t}(A\sin(wt) + B\cos(wt))$$
$$Q(t) = e^{0t}(A\sin(3000t) + B\cos(3000t))$$
$$Q(t) = A\sin(3000t) + B\cos(3000t)$$

Derivative

$$Q'(t) = 3000\,A\cos(3000t) - B\sin(3000t)$$

$$Q(0) = 1\times10^{-2}$$
$$A\sin(3000t) + B\cos(3000t) = 1\times10^{-2}$$
$$A\sin(3000(0)) + B\cos(3000(0)) = 1\times10^{-2}$$
$$A(0) + B(1) = 1\times10^{-2}$$
$$B = 1\times10^{-2}$$

$$Q'(0) = 0$$
$$3000\,A\cos(3000(0)) - B\sin(3000(0)) = 0$$
$$3000\,A\cos(3000(0)) - B\sin(3000(0)) = 0$$
$$3000(A(1) - B(0)) = 0$$

$$\frac{3000A}{3000} = \frac{0}{3000} \qquad A = 0$$

$$Q(t) = A \sin(3000t) + B \cos(3000t)$$
$$Q(t) = 0 \sin(3000t) + (1 \times 10^{-2}) \cos(3000t)$$
$$Q(t) = (1 \times 10^{-2}) \cos(3000t)$$
$$\boxed{Q(t) = 0.01 \cos(3000t)}$$

Formula for the charge on the Capacitor