Reduction of orders.

5.6 question # 5

$$y'' - 2y' + y = 7x^{\frac{3}{2}} e^x$$

$$y' = e^x$$

$y = uy_1 \qquad y = \overset{a}{u}\overset{b}{e^x}$

$a = u \quad b = e^x$

$a' = u' \quad b' = e^x$

$$y' = u'e^x + ue^x$$

$$y'' = u''e^x + u'e^x + u'e^x + ue^x = u''e^x + 2u'e^x + ue^x$$

$$u''e^x + 2u'e^x + ue^x - 2(u'e^x + ue^x) + ue^x$$

$$u''e^x + 2u'e^x + ue^x - 2u'e^x - 2ue^x + ue^x$$

$$\overset{u''e^x}{e^x(u'')} = 7x^{\frac{3}{2}} e^x$$

$$u'' = 7x^{\frac{3}{2}}$$

$$\int u'' = \int 7x^{\frac{3}{2}} dx$$

$$\frac{du}{dx} = \frac{14}{5} x^{\frac{5}{2}} + C_1$$

$$u = \frac{4}{5} x^{\frac{7}{2}} + C_1 x + C_2$$

$$y = ue^x = \left(\frac{4}{5} x^{\frac{7}{2}} + C_1 x + C_2\right) e^x$$

Answer

$$\frac{4x^{\frac{7}{2}}e^x}{5} + C_1 x e^x + C_2 e^x$$