5.6

13)

$$x^2y'' - 3xy' + 4y = 4x^4 \quad \text{—(1)}$$

$$y'' - \frac{3x}{x^2}y' + 4/x^2\, y = \frac{4x^4}{x^2}$$

$$\Rightarrow y'' - \frac{3}{x}y' + \frac{4}{x^2}y = 4x^2 \;;\; y_1 = x^2$$

let, $y = y_1 v(x)$

$= x^2 v(x)$ [be any General Sol]

$y' = x^2 v'(x) + 2x v(x)$

$y'' = x^2 v''(x) + 2x v'(x) + 2x v'(x) + 2x v(x)$

plug in eqn (1); $y'' = x^2 v''(x) + 4x v'(x) + 2x v(x)$

$$\Rightarrow x^2\{x^2 v''(x) + 4x v'(x) + 2x v(x)\} - 3x\{x^2 v'(x) + 2x\, v(x)\} + 4x^2 v(x) = 4x^4$$

$$\Rightarrow x^4 v''(x) + x^3 v'(x) - 2x^2 v(x) + 2x^3 v(x) = 4x^4$$

$$\Rightarrow \cancel{x^4}\{v''(x) + \frac{1}{x}v'(x) + \cancel{\frac{2}{x}v(x)} - \cancel{\frac{2}{x}v(x)}\} = 4\cancel{x^4}$$

$$\Rightarrow v''(x) + \frac{1}{x}v(x) = 4 \quad \text{—(ii)}$$

let, $w = v'(x)$ and

$w'(x) = v''(x)$

So, $w'(x) + \frac{1}{x} w(x) = 4$ — (iii)

Its linear D.E.

Integrating factor:

$\mu(x) = e^{\int 1/x \, dx} = e^{\ln|x|} = x$

$\mu(x) = x$

General soln: $w \cdot \mu(x) = \int 4 \, \mu(x) \, dx$

$w \cdot x = \int 4x \, dx$

$w \cdot x = 2x^2 + c_1$

$w = 2x + c_1/x$

Replacing w by $v'(x)$

$v'(x) = (2x + c_1/x)$

$\frac{dv}{dx} = (2x + c_1/x)$

$\int \frac{dv}{dx} = \int \left\{\left(2x + \frac{c_1}{x}\right)\right\} dx$

$v(x) = \frac{2x^2}{2} + c_1 \ln|x| + c_2$

General solution:

$y(x) = x^2(x^2 + c_1 \ln|x| + c_2)$

Fundamental set of soln: $= \{x^2, \ln|x| x^2\}$.