

5.6 Reduction of order

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$$y = x^2 \quad y'' = 2x$$

$$x^2 y'' - 3xy' + 4y = 0$$

$$y_h = a(x)y'' + b(x)y' + c(x)y = 0$$

$$x^r(r^2 - 4r + 4)$$

Solve for r

$$(r-2)(r-2) = 0$$

$$r = 2$$

$$y = C_1 x^2 + C_2 \ln(x) x^2$$

Finding y_p

$$\frac{x^2 y'' - 3xy' + 4y}{x^2} = y'' - \frac{3y'}{x} + \frac{4y}{x^2} = 4x^2$$

$$y_p = u_1 y_1 + u_2 y_2$$

$$y_p = u_1 y_1 + u_2 y_2$$

$$y_p = \frac{(-2x^2 \ln(x) + x^2)x^2}{\downarrow} + 2x^2 x^2 \ln(x)$$

$$x^2 (-2x^2 \ln(x) + x^2) \quad \checkmark$$

$$\downarrow \quad \quad \quad 2x^4 \ln(x)$$

$$\cancel{2x^4 \ln(x)} + \cancel{(x^4)} + 2x^4 \ln(x)$$

$$y_p = x^4$$

Final

$$y = y_h + y_p$$

$$= \boxed{C_1 x^2 + C_2 \ln(x) x^2 + x^4}$$

General Solution
for one real root

$$y = C_1 x^r + C_2 \ln(x) x^r$$

$$y_1 = x$$

$$y_2 = 2x \ln(x) + x$$

$$\text{Wronskian } w(y_1, y_2) = y_1 y_2 - y_1 y_2$$

$$\overline{u_1} = \cancel{\int -x \ln(x) dx} = x^2 (2x \ln(x) + x)$$

$$-2x^3 \ln(x)$$

$$= x^3$$

$$u_1 = \cancel{\int -x^2 \ln(x) 4x^2} \rightarrow -2x^3 \ln(x) x^2$$

$$u_2 = \cancel{\int x^2 \cdot 4x^2} \rightarrow 2x^2$$