

### 5.6 Reduction of Order

$$13. x^2 y'' - 3xy' + 4y = 4x^4$$

$$\text{Let } x = e^t$$

$$xy' = Dy \quad (D = d/dt)$$

$$x^2 y'' = D(D-1)y$$

$$D(D-1) - 3D + 4 = 4e^{4t}$$

$$D^2 - 4D + 4 = 4e^{4t}$$

$$r^2 - 4r + 4 = 0$$

$$(r-2)(r-2) = 0$$

$$r = 2, 2$$

$$y_c = c_1 e^{2t} + c_2 t e^{2t}$$

$$\begin{aligned} \text{For } Y_p &= \frac{4e^{4t}}{D^2 - 4D + 4} \\ &= \frac{4e^{4t}}{16 - 16 + 4} \\ &= e^{4t} \end{aligned}$$

$$y = (c_1 + c_2 t) e^{2t} + e^{4t}$$

$$= \boxed{c_1 x^2 + c_2 \ln(x) x^2 + x^4}$$