Chapter 6 N°4

Let L be the length that the string is Stretched:
$L = 6$ inches which is $\frac{1}{2}$ ft

For initial displacement, we have $t = 0$, so
$Y(0) = 3$ inches $= \frac{1}{4}$ ft

The given downward speed is equal to 6 in/sec $= \frac{1}{2}$ ft/sec

The equation of motion is: $mY'' + KY = 0$

or

$$Y'' + \frac{K}{m} Y = 0$$

From the equation $mg = K\Delta L$, we can find $\frac{K}{m}$ to complete the second order equation of the form: $Y'' + P(x)Y' + Q(x)Y = 0$
$P(x) = 0$ for spring mass is neglected. So, it is $Y'' + Q(x)Y = 0$

$$\frac{K}{m} = \frac{g}{\Delta L} = \frac{K}{m} = \frac{32}{1/2}$$

$\frac{K}{m} = 64$ Thus: $Y'' + 64Y = 0$
Let $Y = e^{rx}$ and $Y'' = r^2 e^{rx}$

$$r^2 e^{rx} + 64 e^{rx} = 0$$

we factorise e^{rx}: $e^{rx}(r^2 + 64) = 0$

$\frac{0}{e^{rx}} = 0$, so $r^2 + 64 = 0$

$r^2 = -64$

$r = \pm\sqrt{-64} = \pm\sqrt{64} \cdot \sqrt{-1}$

$\sqrt{-1} = i$

$r = \pm 8i$

So the general solution is: $Y(t) = A\cos(8t) + B\sin(8t)$
We know that $Y(0) = \frac{1}{4}$ ft
and $Y'(0) = \frac{1}{2}$ ft/sec

The antiderivative of Y is: $Y'(t) = -8A\cos(8t) + 8B\sin(8t)$
We can apply the initial condition to find A and B:
We know $A = \frac{1}{4}$ and $B = \frac{1/2}{8}$

$B = \frac{1}{16}$

Thus: $Y(t) = \frac{1}{4}\cos(8t) + \frac{1}{16}\sin(8t)$ is the position or displacement of the object.

The formulas for the Frequency(ω), period(T), Amplitude(R), phase angle are respectively:

$\omega = \sqrt{\frac{K}{m}} = \sqrt{64} = 8$ rad/sec

$T = \frac{2\pi}{\omega} = \frac{2\pi}{8} = \frac{\pi}{4}$

$R = \sqrt{A^2 + B^2} = \sqrt{(\frac{1}{4})^2 + (\frac{1}{16})^2}$

$R = \sqrt{\frac{1}{16} + \frac{1}{256}} = \sqrt{\frac{17}{256}} = 0.257$

$\text{PhaseAngle} = \tan^{-1}(\frac{B}{A}) =$

$\text{PhaseAngle} = \tan^{-1}(\frac{4}{16}) = 14.03°$