$y' + 3y = xy^2(y+1)$, $y(0) = 1$

$h = 0.1$, $h = 0.05$, $h = 0.025$ $[0,1]$

Euler's method

For $h = 0.1$

$x_0 = 0$ $\quad y_0 = 1$

$y_{n+1} = y_n + hf(x_n, y_n)$

$y_1 = 1 + 0.1 f(0,1)$

$y_1 = 1 + 0.1(0 \cdot 1^2(1+1) - 3 \cdot 1)$

$y_1 = 1 + 0.1 \cdot (-3) = 0.7$

$y(0.1) = 0.7$

For $h = 0.05$

$y_1 = y_0 + h f(x_0, y_0) =$

$y_1 = 1 + 0.05 f(0,1)$

$y_1 = 1 + 0.05(-3)$

$y_1 = 0.85$

So $y_2 = y_1 + h f(x_1, y_1)$

$y_2 = 0.85 + 0.05 f(0.05, 0.85)$

$y_2 = 0.85 + 0.05(-2.48)$

$y_2 = 0.85 - 0.124$

$y(0.05) = 0.726$

For $h = 0.025$

$y_1 = y_0 + h f(x_0, y_0)$

$y_1 = 1 + 0.025 f(0,1)$

$y_1 = 1 + 0.025(-3)$

$y_1 = 1 - 0.075 = 0.925$

$y_2 = y_1 + hf(x_1, y_1)$

$y_2 = 0.925 + 0.025 f(0.025, 0.925)$

$y_2 = 0.925 + 0.025(-2.734)$

$y_2 = 0.925 - 0.0684$

$y_2 = 0.8567$

$y_3 = y_2 + h f(x_2, y_2)$

$y_3 = 0.8567 + 0.025 f(0.05, 0.8567)$

$y_3 = 0.8567 + 0.025(-2.5)$

$y_3 = 0.8567 - 0.0625$

$y_3 = 0.7942$

$y_4 = y_3 + hf(x_3, y_3)$

$y_4 = 0.7942 + 0.025 f(0.075, 0.7942)$

$y_4 = 0.7942 + 0.025(-2.298)$

$y_4 = 0.7942 - 0.0575$

$y_4 = 0.7368$

Improved Euler's Method

$$y_{n+1} = y_n + \frac{h}{2}\{f(x_n, y_n) + f[x_n + h, y_n + h f(x_n, y_n)]\}$$

$x_0 = 0 \quad y_0 = 1$

For $h = 0.1$

$$y_1 = y_0 + \frac{h}{2}\{f(x_0, y_0) + f[x_0 + h, y_0 + h f(x_0, y_0)]\}$$

$$y_1 = 1 + \frac{0.1}{2}\{f(0,1) + f[0+0.1, 1 + 0.1 f(0,1)]\}$$

$$y_1 = 1 + 0.05\{(-3) + f(0.1, 0.7)\}$$

$$y_1 = 1 + 0.05(-3 - 2.02) = 0.749$$

For $h = 0.05$

$$y_1 = y_0 + \frac{h}{2}\{f(x_0, y_0) + f[x_0 + h, y_0 + h f(x_0, y_0)]\}$$

$$y_1 = 1 + \frac{0.05}{2}\{(-3) + f[0 + 0.05, 1 + 0.05 f(0,1)]\}$$

$$y_1 = 1 + \frac{0.05}{2}\{(-3) + f(0.05, 0.85)\}$$

$$y_1 = 1 + \frac{0.05}{2}(-3 - 2.48) = 1 - 137 = 0.863$$

$$y_2 = y_1 + \frac{h}{2}\{f(x_1, y_1) + f[x_1 + h, y_1 + h f(x_1, y_1)]\}$$

$$y_2 = 0.863 + \frac{0.05}{2}\{f(0.05, 0.863) + f[0.05 + 0.05, 0.863 + 0.05 f(0.05, 0.863)]\}$$

$$y_2 = 0.863 + \frac{0.05}{2}\{(-2.52) + f(0.1, 0.737)\}$$

$$y_2 = 0.863 + \frac{0.05}{2}(-4.64) = 0.863 - 0.116$$

$$y_2 = 0.747$$

For $h = 0.025$

$$y_1 = y_0 + \frac{h}{2}\{f(x_0, y_0) + f[x_0 + h, y_0 + h f(x_0, y_0)]\}$$

$$y_1 = 1 + \frac{0.025}{2}\{f(0,1) + f[0 + 0.025, 1 + 0.025 f(0,1)]\}$$

$$y_1 = 1 + \frac{0.025}{2}\{(-3) + f(0.025, 0.925)\}$$

$$y_1 = 1 + \frac{0.025}{2}(-3 - 2.734) = 1 - 0.072$$

$$y_1 = 0.928$$

$$y_2 = y_1 + \frac{h}{2}\{f(x_1, y_1) + f[x_1 + h, y_1 + h f(x_1, y_1)]\}$$

$$y_2 = 0.928 + \frac{0.025}{2}\{f(0.025, 0.928) + f[0.025 + 0.025, 0.928 + 0.025 f(0.025, 0.928)]\}$$

$$y_2 = 0.928 + \frac{0.025}{2}\{(-2.7) + f(0.05, 0.861)\}$$

$$y_2 = 0.928 + \frac{0.025}{2}(-5.21) = 0.928 - 0.065$$

$$y_2 = 0.863$$

$$y_3 = y_2 + \frac{h}{2}\{f(x_2, y_2) + f[x_2 + h, y_2 + h f(x_2, y_2)]\}$$

$$y_3 = 0.863 + \frac{0.025}{2}\{f(0.05, 0.863) + f[0.05 + 0.025, 0.863 + 0.025 f(0.05, 0.863)]\}$$

$$y_3 = 0.863 + \frac{0.025}{2}\{(-2.52) + f(0.075, 0.8)\} = 0.863 + \frac{0.025}{2}((-2.52) - 2.31)$$

$$y_3 = 0.863 - 0.06 = 0.8$$

$$y_4 = y_3 + \frac{h}{2}\{f(x_3, y_3) + f[x_3 + h, y_3 + h f(x_3, y_3)]\}$$

$$y_4 = 0.8 + \frac{0.025}{2}\{f(0.075, 0.8) + f[0.075 + 0.025, 0.8 + 0.025 f(0.075, 0.8)]\}$$

$$y_4 = 0.8 + \frac{0.025}{2}\{(-2.31) + f(1, 0.78)\} = 0.8 + \frac{0.025}{2}((-2.31) - 1.257)$$

$$y_4 = 0.8 - 0.0446 = 0.755$$