Solving secondary order equations using reduction of order and variation of parameters

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5.7 #1) $y'' + 9y = \tan 3x$

$y'' + 9y \Rightarrow r^2 + 9 = 0$

$r^2 = -9 \Rightarrow r = \pm 3i$

$r = 3i, \; r = -3i$

$y = c_1 \cos(3x) + c_2 \sin(3x)$

$y_p = u_1 y_1 + u_2 y_2$

$u_1 = \int -\frac{y_2\, g(x)}{W(y_1, y_2)}\, dx$

$u_2 = \int -\frac{y_1\, g(x)}{W(y_1, y_2)}\, dx$

$y_1 = \cos(3x)$

$y_2 = \sin(3x)$

$y_1' = (\cos(3x))' = -\sin(3x)\cdot(3)$

$y_2' = (\sin(3x))' = \cos(3x)\cdot 3$

$W(y_1, y_2) = y_1 y_2' - y_1' y_2$

$= \cos(3x)\cos(3x)\cdot 3 - (-\sin(3x)\cdot 3)\sin(3x)$

$W = 3$

$g(x) = \tan 3x$

$u_1 = \int -\frac{y_2\, g(x)}{W(y_1, y_2)}\, dx$

$= \int -\frac{\sin(3x)\tan(3x)}{3}\, dx = -\frac{1}{3}\int \overset{u}{\sin(3x)}\, \overset{v'}{\tan(3x)}\, dx$

$u = \tan(3x)$

$u' = (\tan(3x))' = \sec^2(3x)\cdot 3$

$v' = \sin(3x)$

$v = \int \sin(3x)\, dx = -\frac{1}{3}\cos(3x)$

integration by parts #1

$\int uv' = uv - \int u'v$

$\tan(3x)\left(-\frac{1}{3}\cos(3x)\right) - \int \sec^2(3x)\cdot 3\left(-\frac{1}{3}\cos(3x)\right) dx$

$= -\frac{1}{3}\left(-\frac{1}{3}\tan(3x)\cos(3x) - \int -\sec^2(3x)\cos(3x)\, dx\right)$

Solving integral

$\int -\sec^2(3x)\cos(3x)\, dx)$

$= -\int \sec^2(3x)\cos(3x)\, dx)$ Trig identity $\sec(x) = \frac{1}{\cos(x)}$

$= -\int \sec(3x)\, dx$

u-sub

$u = 3x$

$du = 3\, dx$

$dx = \frac{1}{3} du$

$= \int \sec(u)\frac{1}{3} du$

$= -\frac{1}{3}\cdot\int \sec(u)\, du$

$= -\frac{1}{3}\ln|\tan(u) + \sec(u)|$

$= -\frac{1}{3}\ln|\tan(3x) + \sec(3x)|$

$= -\frac{1}{3}\left(-\frac{1}{3}\tan(3x)\cos(3x) - \left(-\frac{1}{3}\ln|\tan(u) + \sec(u)|\right)\right)$

$= -\frac{1}{3}\left(-\frac{1}{3}\tan(3x)\cos(3x) + \frac{1}{3}\ln|\tan(u) + \sec(u)| + C = u_1\right.$

$u_2 = \int \frac{y_1\, g(x)}{W}\, dx$

$= \int \frac{\cos(3x)\tan(3x)}{3}\, dx$

$= \frac{1}{3}\cdot\int \cos(3x)\tan(3x)\, dx$

u Sub

$u = 3x$

$du = 3\, dx$

$dx = \frac{1}{3} du$

$\frac{1}{3}\cdot\int \cos(u)\tan(u)\frac{1}{3} du$

$= \frac{1}{3}\cdot\frac{1}{3}\cdot\int \cos(u)\tan(u)\, du$

$= \frac{1}{9}\cdot\int \sin(u)\, du$

$= \frac{1}{9}\cdot(-\cos(3x))$

$u_2 = -\frac{1}{9}\cos(3x)$

$y_p = u_1 y_1 + u_2 y_2$

$y_p = \left(-\frac{1}{3}\left(-\frac{1}{3}\tan(3x)\cos(3x) - \left(-\frac{1}{3}\ln|\tan(u) + \sec(u)|\right)\right)\right)\cos(3x)$

$+ \left(-\frac{1}{9}\cos(3x)\right)\sin(3x)$

$= \frac{-2\cos(3x)\left(-\tan(3x)\cos(3x) + \ln|\tan(3x) + \sec(3x)|\right) - \sin(6x)}{18}$

Final Answer everything together

$y = C_1\cos(3x) + C_2\sin(3x) + \frac{-2\cos(3x)\left(-\tan(3x)\cos(3x) + \ln|\tan(3x) + \sec(3x)|\right) - \sin(6x)}{18}$