Monday, September 14, 2020

Project 1 Sec 4.3

General Notes on Elementary Mechanics (pg. 151):

Newton's Second Law of Motion

$$F = ma$$

Standard Forms:

Time (t)

Displacement/ Position (y):	$y = y(t)$		
Velocity (v):	$v = v(t)$	$v = y'$	
Acceleration (a)($\frac{dv}{dt}$):	$a = a(t)$	$a = v'$	$a = y''$
Force (F):	$F = m\ a$		
Force of Gravity/ Weight (F_g):	$F_g = m\ g$		
Resistive Force (k):	$k = m\ a$		
Acceleration due to Gravity (g):	$9.8\ m/s^2$ (mks); $980\ cm/s^2$ (cgs); $32\ f/s^2$ (British)		

Use Eq. (4.3.4):

$$F = - mg + F_1$$

Where mg is the force due to gravity and F_1 is the resisting force of the medium (pg. 152).

If the object is moving upward ($v>=0$), the resisting force is downward

$$F_1 = -k\,|v| = -kv$$

Rewrite Eq. (4.3.4) as (4.3.5):

$$F = - mg - kv$$

Regardless of the sign of the velocity. From Newton's second law of motion:

$$F = ma = mv'$$

so (4.3.5) yields:

$$m\,v' = - m\,g - k\,v$$

Problem 5:

A stone weighing 1/2 lb is thrown upward from an initial height of 5 ft with an initial speed of 32 ft/s. Air resistance is proportional to speed, with k = 1/128 lb-s/ft. Find the maximum height attained by the stone.

Using Form:

$$m \; * \; v' \; = -m\,g \; - \; k\,v$$

$$\frac{1}{64} lb \; * \; v' \; = - \; \frac{1}{64} lb * \; 32\,ft/s^2 - \frac{1}{128}\frac{lb - s}{ft} * \; v$$

Simplify/Rewrite:

$$v' \; + \frac{1}{2} v = -32$$

Solve:

$$v' \; + \frac{1}{2} v = 0$$

$$\int \frac{dv}{dt} + \frac{1}{2} \; v \; dt \; = 0$$

$$\int \frac{1}{v} \; dv \; = \int \frac{1}{2} dt$$

$$ln|v| \; = -\frac{t}{2} + c$$

$$v \; = \; e^{-\frac{t}{2}}$$

With the above solution to the complementary equation, the solution for this equation are as follows:

$$v \; = \; u * e^{-\frac{t}{2}}$$

Where:

$$u' * e^{-\frac{t}{2}} \; = -32$$

So:

$$u' = -32 * e^{-\frac{t}{2}} \; = -32$$

Hence:

$$u = -64e^{-\frac{t}{2}} + c$$

So:

$$v = u * e^{-\frac{t}{2}} = -64 + c * e^{-\frac{t}{2}}$$

Let Vo= 32 ft/s, t=0

$$32 = -64 + c * e^{-\frac{0}{2}}$$

$$c = 96$$

When v=0, find t:

$$0 = -64 + 96 * e^{-\frac{t}{2}}$$

$$t = 2\,ln(\frac{2}{3}) = .811$$

Position = y(t)= y

v=y'

y=$\int$v

$$y(t) = \int -64 + 96 * e^{-\frac{t}{2}}$$

$$y(t) = -64t - 192 * e^{-\frac{t}{2}} + c$$

When t=0 and y(t) = 5:

$$5 = -64(0) - 192 * e^{-\frac{0}{2}} + c$$

$$5 = -192 + c$$
$$c = 197$$

Position after t=.811:

$$y(.811) = -64(.811) - 192 * e^{-\frac{.811}{2}} + 197 = 17.1005ft$$

Solution: 17.10 ft