

Project 1 Part 1

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Differential Equations

MAT 2680 Section OL67

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Summer of Thought Process and Approach to the Problem

Solving Real Life Applications Of Differential Equations

Group 1 Section 4.1 Number 17

Problem: A tank is empty at $t = 0$. Water is added to the tank at a rate of 10 gal/min, but it leaks out at a rate equal to the number of gallons in the tank. What is the smallest capacity the tank can have if this process is to continue forever?

Approach: I approached this problem understanding that there is a tank and a tank has volume but the problem has no relation solving volume. But the problem deals with water and how it's being displaced. I gave the input, the amount of water per gallon *going into the tank*, the variable x , and the output, the amount of water per gallon *leaving the tank* the variable $x(t)$ this can also be the amount of water at any given point as this is a function of t (time). As this will also behave as a dependent variable being reliable on the independent variable the input amount of water going into the tank. We setup a formula

$$\left(\frac{d}{dt} \right) x = x - 10$$

I set $dx/dt = 0$ as this is telling us that at 0 seconds there is no water and shows this is the starting point. Solving algebraically we add $(+ 10)$ on each side, to arrive at $x = 10$.

With this information we can tell, If x is less than 10, the water level in the tank will increase. If x is Greater than 10, the water level in the tank will decrease. When x equals 10 the tank is at equilibrium meaning the tank maintains 10 gallons of water per minute.

Piecing everything together with the prompted question, What is the smallest capacity the tank can have if this process is to continue forever? I have arrived at the answer 10 gallon per min is the stable amount of water to continue this process forever.

Handwritten work and calculations

$t=0$ water = AOC to tank (water)

$$X(t) = \text{out} \quad X = IN = 10 \text{ gal/min}$$

$$= \frac{dx}{dt} = (\text{rate in}) - (\text{rate out})$$

$$= \frac{dx}{dt} = X - 10 \quad \frac{dx}{dt} = 0 \Rightarrow t_0 = 0$$

$$0 = X - 10 \Rightarrow X = 10$$

$$X < 10, \quad \frac{dx}{dt} = 10 - X > 0$$

if X is less than 10, the water level (in the tank) will increase.

if X is greater than 10, the water level (in the tank) will decrease

When $X = 10$ the tank is at equilibrium, meaning the tank maintains 10 gallon of water per min.