

$A =$  initial amount  
 $k =$  is decay rate

$$A(t) = A e^{kt}$$

$$A(t_p) = \frac{A}{p}, \Rightarrow \frac{A}{p} = A e^{kt_p} \Rightarrow \frac{1}{p} = e^{kt_p}$$

$$A(t_q) = \frac{A}{q}, \Rightarrow \frac{A}{q} = A e^{kt_q} \Rightarrow \frac{1}{q} = e^{kt_q}$$

$$\begin{aligned} \frac{1}{p} = e^{kt_p} &\Rightarrow \ln \frac{1}{p} = \ln(e^{kt_p}) \Rightarrow \ln 1 - \ln p = kt_p \\ &\Rightarrow -\ln p = kt_p \Rightarrow \boxed{k = \frac{-\ln p}{t_p}} \end{aligned}$$

$$\begin{aligned} \frac{1}{q} = e^{kt_q} &\Rightarrow \ln \frac{1}{q} = \ln(e^{kt_q}) \Rightarrow -\ln q = kt_q \\ &\Rightarrow \boxed{k = \frac{-\ln q}{t_q}} \end{aligned}$$

Therefore

$$\frac{t_p}{t_q} = \frac{\ln p}{\ln q}$$