(a) Let $\vec{u} = \langle 2, 1, 5 \rangle$. Find vectors $\vec{v}$ and $\vec{w}$ so that the set $\{\vec{u}, \vec{v}, \vec{w}\}$ forms an orthogonal basis of $\mathbb{R}^3$.

- Let $\vec{v} = \langle a, b, c \rangle$
- Let $\vec{w} = \langle x, y, z \rangle$

$$2a + b + 5c = 0$$

- Let $a, c = 1, 0$

$$\rightarrow b = -2$$

$$\boxed{\vec{v} = \langle 1, -2, 0 \rangle}$$

$$\left.\begin{aligned} x - 2y &= 0 \\ 2x + y + 5z &= 0 \end{aligned}\right\}$$

- Let $x = 2$

$$\rightarrow y = 1$$

$$\rightarrow z = -1$$

$$\boxed{\vec{w} = \langle 2, 1, -1 \rangle}$$

(b) Normalize your answer from part (a) to find a orthonormal basis of $\mathbb{R}^3$

$$\|\vec{v}\| = \sqrt{(1)^2 + (2)^2 + (0)^2} = \sqrt{5}$$

$$\|\vec{w}\| = \sqrt{(2)^2 + (1)^2 + (1)^2} = \sqrt{6}$$

$$\|\vec{u}\| = \sqrt{(2)^2 + (1)^2 + (5)^2} = \sqrt{30}$$

$$\left\{ \begin{bmatrix} \frac{2}{\sqrt{30}} \\ \frac{1}{\sqrt{30}} \\ \frac{5}{\sqrt{30}} \end{bmatrix}, \begin{bmatrix} \frac{1}{\sqrt{5}} \\ \frac{-2}{\sqrt{5}} \\ 0 \end{bmatrix}, \begin{bmatrix} \frac{2}{\sqrt{6}} \\ \frac{1}{\sqrt{6}} \\ \frac{-1}{\sqrt{6}} \end{bmatrix} \right\}$$