

Name:

Practice Final Exam

1. True/false

- 1) Cramer's rule can be used on $Ax=b$ when $\det(A)=0$.
- 2) It's possible for a basis of \mathbb{R}^n to contain the zero vector.
- 3) Vectors in an orthogonal set are always linearly independent.
- 4) The row echelon form of a square matrix is unique.
- 5) For a $n \times n$ matrix A , if A is invertible, then A is diagonalizable.

2. Example

- 1) Provide an example of two parallel vectors in an \mathbb{R}^2 space.
- 2) Give an example of an invertible matrix A such that $A = A^{-1}$.
- 3) Give one of the standard basis of \mathbb{R}^3 .
- 4) Give an example of a transformation $T: \mathbb{R}^2 \rightarrow \mathbb{R}^2$ that is not linear.
- 5) Give an example of a 2×2 matrix that don't have real eigenvalues.

3. Let $A = \begin{bmatrix} 2 & -5 & 5 \\ 0 & 4 & 3 \end{bmatrix}$, $B = \begin{bmatrix} 8 & -6 \\ -4 & -1 \\ 2 & 3 \end{bmatrix}$, then:

- 1) $AB = _$
- 2) $BA = _$
- 3) $\det(AB) = _$
- 4) $\det(BA) = _$

4. Use the Cramer's Rule to find values for x , y , and z that satisfy the following systems

$$\begin{aligned} -5x + 6y + 4z &= 0 \\ -7x - 8y + 2z &= -20 \\ 2x + 9y - z &= 12 \end{aligned}$$

5. Find the basis of the subspace \mathbb{R}^4 that consists of all vectors perpendicular to both

$$\begin{bmatrix} 1 \\ 0 \\ 1 \\ -9 \end{bmatrix} \text{ and } \begin{bmatrix} 0 \\ 1 \\ -8 \\ -1 \end{bmatrix}.$$

6. The matrix $A = \begin{bmatrix} -11 & 3 \\ -6 & -2 \end{bmatrix}$ has eigenvalues -5 and -8 . Find its eigenvectors.

7. Given that the matrix A has eigenvalues $\lambda_1 = 4$ with corresponding eigenvector $\vec{v}_1 = \begin{bmatrix} -1 \\ -2 \end{bmatrix}$ and $\lambda_2 = -4$ with corresponding eigenvector $\vec{v}_2 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$, find A .