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Mat 1575

Calc 2

Test #2 review

Find the Taylor polynomial of degree  $n=4$  for  $x$  near the point  $a = \frac{\pi}{8}$  for the function  $(\sin(4x))$

Given  $f(x) = \sin(4x)$ , at  $a = \frac{\pi}{8}$

$$f'(x) = 4 \cos(4x)$$

$$f''(x) = -4^2 \sin(4x)$$

$$f'''(x) = -(4)^3 \cos(4x)$$

$$f^{(4)}(x) = 4^4 \sin(4x)$$

$$f(a) = \sin(4 \cdot \frac{\pi}{8}) = 1$$

$$f'(a) = 0 \quad f''(a) = -4^2, \quad f'''(a) = 0, \quad f^{(4)}(a) = 4^4$$

The Taylor poly at  $n=4$  at  $x=a$  would be

$$P_4(x) = f(a) + \frac{x-a}{1!} f'(a) + \frac{(x-a)^2}{2!} f''(a) + \frac{(x-a)^3}{3!} f'''(a) + \frac{(x-a)^4}{4!} f^{(4)}(a)$$

$$P_4(x) = f(\frac{\pi}{8}) + \frac{x - \frac{\pi}{8}}{1!} f'(\frac{\pi}{8}) + \frac{(x - \frac{\pi}{8})^2}{2!} f''(\frac{\pi}{8}) + \frac{(x - \frac{\pi}{8})^3}{3!} f'''(\frac{\pi}{8}) + \frac{(x - \frac{\pi}{8})^4}{4!} f^{(4)}(\frac{\pi}{8})$$

$$= 1 + 0 + \frac{(x - \frac{\pi}{8})^2}{2!} \times (-4)^2 + 0 + \frac{(x - \frac{\pi}{8})^4}{4!} \times (4)^4$$

$$P_4(x) = 1 - 8 \left(x - \frac{\pi}{8}\right)^2 + \frac{32}{3} \left(x - \frac{\pi}{8}\right)^4$$