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Improper Integrals: Problem 22

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(1 point) **Library/UCSB/Stewart5_7_8/Stewart5_7_8_59.pg**

(a) Find the values of p for which the following integral converges:

$$\int_0^1 x^p \ln(x)\,dx$$

Input your answer by writing it as an interval. Enter brackets or parentheses in the first and fourth blanks as appropriate, and enter the interval endpoints in the second and third blanks. Use INF and NINF (in upper-case letters) for positive and negative infinity if needed. If the improper integral diverges for all p, type an upper-case "D" in every blank.

Values of p are in the interval

For the values of p at which the integral converges, evaluate it. Integral =

Note: *You can earn partial credit on this problem.*

Edit3

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$$\int_0^1 x^{-3}\ln(x)\,dx$$

$$= \lim_{a\to 0^+}\int_a^1 x^{-3}\ln(x)\,dx$$

$$= \lim_{a\to 0^+}\left(-\frac{\ln(x)}{2x^2} - \frac{1}{4x^2}\right)\Bigg|_a^1$$

$$\int x^{-3}\ln(x)\,dx$$

$$= \int \frac{1}{x^3}\ln(x)\,dx$$

$u = \ln(x)$ $\quad p = -3$

$du = \frac{1}{x}dx$

$v = -\frac{1}{2}x^{-2}$

$dv = \frac{1}{x^3}dx$

$v\,du$

$$= -\frac{1}{2x^2}\ln(x) - \int -\frac{1}{2}x^{-2}\cdot\frac{1}{x}\,dx$$

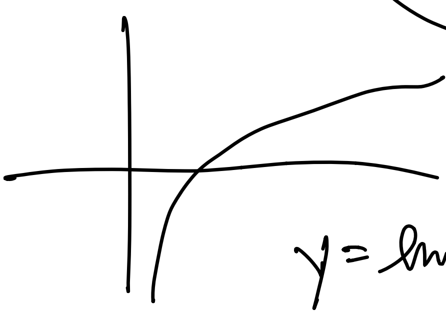
$$= -\frac{\ln(x)}{2x^2} + \frac{1}{2}\int x^{-3}\,dx$$

$$= -\frac{\ln(x)}{2x^2} - \frac{1}{2\cdot 2}x^{-2} + C$$

$$= -\frac{\ln(x)}{2x^2} - \frac{1}{4x^2} + C$$

$$= \lim_{a\to 0^+}\left(-\frac{\ln(1)}{2\cdot 1^2} - \frac{1}{4\cdot 1^2} - \left(-\frac{\ln(a)}{2a^2} - \frac{1}{4a^2}\right)\right)$$

$$= \frac{0}{2} - \frac{1}{4} - \left(-\frac{(-\infty)}{2(0^2)} - \frac{1}{4\cdot 0^2}\right)$$



$y = \ln(x)$

limit does not exist

So Integral diverges

$$\int_0^1 x^p \ln(x)\,dx$$

$$= \lim_{a\to 0^+}\int_a^1 x^p \ln(x)\,dx$$

$$\int x^p \ln(x)\,dx$$

$u = \ln(x)$

$du = \frac{1}{x}dx$

$v = \frac{1}{p+1}x^{p+1}$

$dv = x^p dx$

$v\,du$

$$= \lim_{a \to 0^+} \left(\frac{\ln(x)\, x^{p+1}}{p+1} - \frac{x^{p+1}}{(p+1)^2} \right) \Bigg|_a$$

$$= \frac{\ln(x)\, x^{p+1}}{p+1} - \int \frac{1}{p+1} x^{p+1} \cdot \frac{1}{x}\, dx$$

$$= \frac{\ln(x)\, x^{p+1}}{p+1} - \frac{1}{p+1} \int x^p\, dx$$

$$= \frac{\ln(x)\, x^{p+1}}{p+1} - \frac{1}{p+1} \cdot \frac{x^{p+1}}{p+1} + C$$

$$= \lim_{a \to 0^+} \left(\frac{\ln(1) \cdot 1^{p+1}}{p+1} - \frac{1^{p+1}}{(p+1)^2} \right) - \left(\frac{\ln(a) \cdot a^{p+1}}{p+1} - \frac{a^{p+1}}{(p+1)^2} \right)$$

$$= \frac{0 \cdot 1}{p+1} - \frac{1}{(p+1)^2} - \underline{\qquad\qquad}$$

consider

$$\lim_{a \to 0^+} \ln(a) \cdot a^{p+1}$$

?

examples

① $\lim \quad \ln(a) \cdot a^{-1+1}$

$$\lim_{a\to 0^+} \ln(a)\cdot a$$

$p=-1$

$$=\lim_{a\to 0^+} \ln(a)$$

$$= -\infty$$

does not exist

② $$\lim_{a\to 0^+} \ln(a)\, a^{0+1}$$

$p=0$

$$=\lim_{a\to 0^+} \ln(a)\cdot a$$

$\ln(a)\to -\infty$

$a \to 0$

$$=\lim_{a\to 0^+} \frac{\ln(a)}{\frac{1}{a}}$$

$\ln(a)\to -\infty$

$\frac{1}{a} \to \infty$

L'hôpital's rule

$$=\lim_{a\to 0^+} \frac{\frac{1}{a}}{-\frac{1}{a^2}}$$

$$=\lim_{a\to 0^+} \left(\frac{1}{a}\cdot\left(-\frac{a^2}{1}\right)\right)$$

$$=\lim_{a\to 0^+} (-a)$$

$$= 0$$

$$\lim_{a\to 0^+} \ln(a)\, a^{p+1}$$

does not exist

$p < -1$

$p+1 < 0$

$\lim_{a\to 0^+} a^{p+1}$ does not exist

if $p > -1$

$$\lim_{a\to 0^+} \ln(a)\, a^{p+1}$$

$$= \lim_{a\to 0^+} \frac{\ln(a)}{\frac{1}{a^{p+1}}}$$

$a^{p+1} \to 0$

$\frac{1}{a^{p+1}} \to \infty$

$$= \lim_{a\to 0^+} \frac{\frac{1}{a}}{\frac{-(p+1)}{a^{p+2}}}$$

$$= \lim_{a\to 0^+} \left(\frac{1}{a}\right)\left(\frac{a^{p+2}}{-p+1}\right)$$

$$= \lim_{a \to 0^+} \frac{a^{p+1}}{-p+1}$$

$= 0$ exists

Consider

$$\lim_{a \to 0^+} a^{p+1}$$

$= 0$

exists

if $p > -1$

$a^{0+1} = a$

$a^{1+1} = a^2$

$\vdots$

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so . . . .

$$= \lim_{a \to 0^+} \left(\frac{\ln(1) \cdot 1^{p+1}}{p+1} - \frac{1^{p+1}}{(p+1)^2} \right) - \left(\frac{\ln(a) \cdot a^{p+1}}{p+1} - \frac{a^{p+1}}{(p+1)^2} \right)$$

exists if $p > -1$

does not exist if $p \leq -1$

integral converges if p is in the interval

$(-1, \infty)$