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root: let $\rho = \lim_{n\to\infty} \sqrt[n]{|a_n|}$, if $\rho < 1$ converges
$\rho > 1$ diverges

Divergence: if $\lim_{n\to\infty} a_n \neq 0$, then $\sum_{n=0}^{\infty} a_n$ diverges

Geometric: if $\sum_{n=1}^{\infty} ar^{n-1}$, then $|r| < 1$ converges
$|r| \geq 1$ diverges

Integral: if $\int_1^{\infty} f(x)dx$ converges/diverges, then so does $\sum_{n=1}^{\infty} a_n$ where $a_n = f(n)$

Comparison: if $0 \leq a_n \leq b_n$ for all $n \geq 0$,

then if $\sum_{n=0}^{\infty} a_n$ diverges, so does $\sum_{n=0}^{\infty} b_n$

if $\sum_{n=0}^{\infty} b_n$ converges, so does $\sum_{n=0}^{\infty} a_n$

Limit Comparison: let $\lim_{n\to\infty} \frac{a_n}{b_n} = L$,

if $L = 0$ and $\sum_{n=0}^{\infty} b_n$ converges, so does $\sum_{n=0}^{\infty} a_n$

$L = \infty$ and $\sum_{n=0} b_n$ diverges, so does $\sum_{n=0}^{\infty} b_n$

else they behave the same

Alternating: if $\sum_{n=1}^{\infty} a_n$ has
1. $a_n \geq 0$
2. $a_n \geq a_{n+1}$
3. $\lim_{n\to\infty} a_n = 0$

then $\sum_{n=1}^{\infty} (-1)^n a_n$ converges

conditional: $\sum_{n=1}^{\infty} |a_n|$ diverges, but $\sum_{n=0} a_n$ converges

ratio: let $\lim_{n\to\infty} \left| \frac{a_{n+1}}{a_n} \right| = \rho$, then if $\rho < 1$ converges
$\rho > 1$ diverges