Russell

HOT Topic Poirier
Assignment Weekly_HOT_Topic due 08/29/2022 at 05:25pm EDT

MAT1575-F22-Poirier-D030

Problem 1. (1 point)
Determine whether the integral is divergent or convergent. If it is convergent, evaluate it. If it is divergent, enter "divergent."
$\int_7^{\infty} e^{-3x}dx =$ ______

Problem 2. (1 point)
Determine whether the following series converges or diverges. In your written work, state any convergence tests you used to determine your answer.

$\sum_{n=1}^{\infty} \frac{6n^6}{4n^6+4}$

The series

- converges
- diverges

Problem 3. (1 point)
Evaluate the indefinite integral.
$\int \frac{4x^2-17x+18}{x(x-3)^2}dx =$ ______

Problem 4. (1 point)
Find the area of the region enclosed by the graphs of $y = x^2 + 9x, y = 7x+3$

Area = ______

Problem 5. (1 point)
Find the center, radius of convergence and interval of convergence for the power series $\sum_{n=1}^{\infty} \frac{(-1)^n(x+4)^n}{n \cdot 8^n}$

Center: $x =$ ______
Radius of Convergence: ______
Interval of Convergence (use interval notation): ______

Problem 6. (1 point)
Find the volume of the solid obtained by rotating the region bounded by the graphs of $y = 14-x, y = 3x+10, x = -3$ about the x-axis.

Volume = ______

Problem 7. (1 point)
Evaluate the indefinite integral:

$\int x^2 \ln(x)\, dx =$ ______

Problem 8. (1 point)
Evaluate the definite integral:

$\int_0^1 -7x(3-x^2)^5\, dx =$ ______

Problem 9. (1 point)
Determine whether the following series converges absolutely, converges conditionally, or diverges. In your written work, state any convergence tests you used to determine your answer.

$\sum_{n=1}^{\infty} (-1)^n 14^{-n}$

The series

- converges absolutely
- converges conditionally
- diverges

Problem 10. (1 point)
Find the Taylor polynomial of degree 3 for the function $f(x) = e^{-6x}$, centered at $a = -2$.

$T_3(x) =$ ______

Problem 11. (1 point)
Evaluate the indefinite integral:

$\int \frac{\sqrt{x^2-36}}{x^4}dx =$ ______

1)

$$\lim_{b\to\infty}\int_7^b e^{-3x}\,dx$$

$$\lim_{b\to\infty}\left[-\frac{e^{-3x}}{3}\right]_7^b$$

$$=\lim_{b\to\infty}\frac{-e^{-3b}}{3}+\frac{e^{-3(7)}}{3}$$

$$\cancel{-\frac{1}{3e^{3b}}}+\boxed{\frac{1}{3e^{21}}}$$

3) $$\int\frac{4x^2-17x+18}{x(x-3)^2}\,dx \rightarrow \frac{A}{x}+\frac{B}{x-3}+\frac{C}{(x-3)^2}$$

$$A(x-3)^2+Bx(x-3)+Cx=4x^2-17x+18$$

$$\cancel{Ax^2}-A6x+\cancel{9A}+\cancel{Bx^2}-3Bx+Cx=4x^2-17x+18$$

$A+B=4$ $\quad 9A=18$ $\quad -12x-6x+Cx=-17x$

$B=2$ $\quad \leftarrow$ $\quad A=2$ $\quad C=1$

$$=2\ln|x|+2\ln|x-3|-\frac{1}{x-3}+C$$

5)

$$\sum_{n=1}^{\infty} \frac{(-1)^n (x+4)^n}{n 8^n}$$

$$\rho = \lim_{n\to\infty} \left| \frac{(-1)^{\cancel{n+1}} (x+4)^{\cancel{n+1}}}{n+1 \; 8^{\cancel{n+1}}} \cdot \frac{n \cancel{8^n}}{\cancel{(-1)^n} \cancel{(x+4)^n}} \right|$$

$$= \left| \frac{-(x+4)}{8} \right|$$

$\rho < 1$:

$$|-x-4| < 8$$

$$-8 < -x-4 < 8$$

$$4 > x > -12$$

bounds:

-12: $\sum_{n=1}^{\infty} \frac{\cancel{(-1)^n} \cancel{(-1)^n} \cancel{8^n}}{n \cancel{8^n}}$ diverges

4: $\sum_{n=1}^{\infty} \frac{(-1)^n \cancel{8^n}}{n \cancel{8^n}}$ converges

$(-12, 4]$

$R = 8$

7)

$$\cancel{\begin{array}{c|c} & \ln x & x^2 \\ \hline + & \frac{1}{x} & \frac{x^3}{3} \\ - & \frac{-1}{x^2} & \frac{x^4}{12} \end{array}}$$

$\int x^2 \ln(x)dx$ let $u = \ln x$ $\quad v = \frac{x^3}{3}$

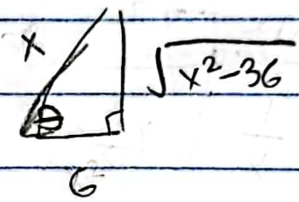
$du = \frac{dx}{x}$ $\quad dv = x^2 dx$

$$= \frac{x^3 \ln x}{3} - \int \frac{x^2}{3} dx$$

$$= \frac{x^3 \ln x}{3} - \frac{x^3}{9} + C$$

(1)

$$\int \frac{\sqrt{x^2-36}}{x^4}$$



$$= \int \frac{\sqrt{36(\sec^2\theta-1)}}{6^4\sec^4\theta}\, 6\sec\theta\tan\theta\, d\theta$$

let $x = 6\sec\theta$

$dx = 6\sec\theta\tan\theta\, d\theta$

$$= \int \frac{6\tan^2\theta}{6^3\sec^3\theta}\, d\theta$$

$$= \frac{1}{36}\int \sin^2\theta\cos\theta\, d\theta$$

let $u = \sin\theta$

$du = \cos\theta\, d\theta$

$$= \frac{1}{36}\int u^2\, du$$

$$= \frac{\sin^3\theta}{108} + C$$

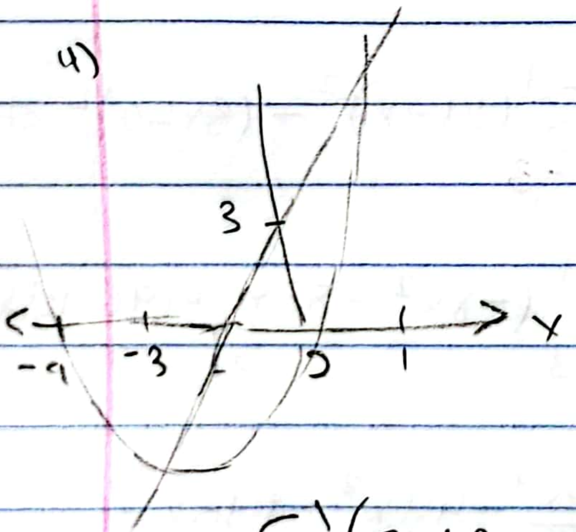
$$= \frac{(x^2-36)^{\frac{3}{2}}}{6x^3} + C$$

2) $$\sum_{n=1}^{\infty} \frac{6n^6}{4n^6+4}$$

$$\lim_{n\to\infty} \frac{6n^6}{4n^6+4} = \frac{3}{2} \neq 0$$

diverges by divergence test

4)



$$x^2+9x = 7x+3$$
$$x^2+2x-3=0$$
$$(x+3)(x-1)=0$$
$$x=1,-3$$

$$\int_{-3}^{1} (7x+3-x^2-9x)\,dx$$

$$= \left[\frac{-x^3}{3} - x^2 + 3x\right]_{-3}^{1}$$

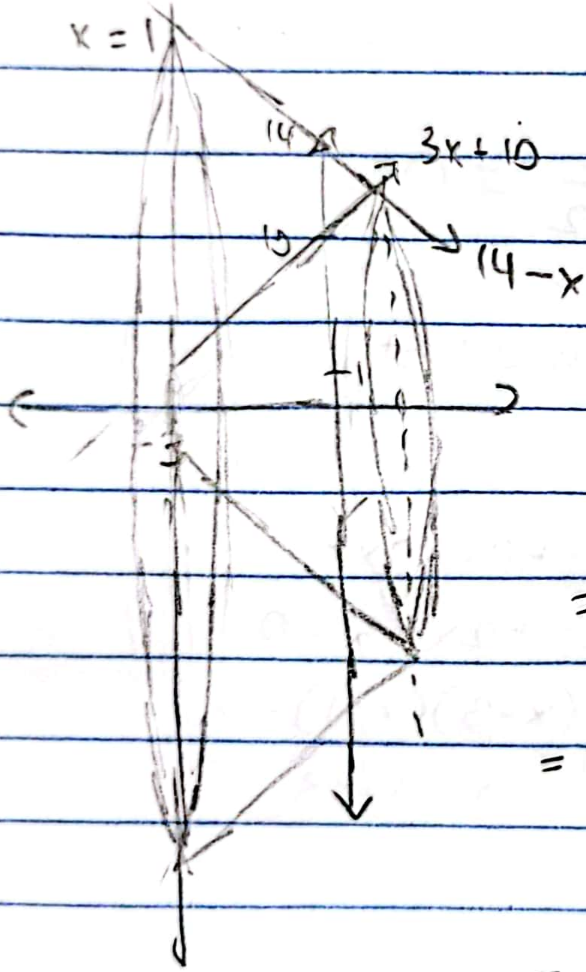
$$= \frac{-1}{3} - 1 + 3 - (9-9-9)$$

$$= 10\tfrac{2}{3}$$

6) $14 - x = 3x + 10$

$4 = 4x$

$x = 1$



$x^2 - 28x + 296$
$-(9x^2 + 60x + 100)$

$= \pi \int_{-3}^{1} (14-x)^2 - (3x+10)^2 \, dx$

$= \pi \int_{-3}^{1} \left(-8x^2 - 88x + 196\right) dx$

$= \pi \left[\frac{-8x^3}{3} - 44x^2 + 196x \right]_{-3}^{1}$

$$\begin{array}{r} 27 \\ \times\ 8 \\ \hline 216 \end{array}$$

$= \pi \left[\frac{-1}{3} - 44 + 196 - \left(\frac{216}{3} - 396 - 588 \right) \right]$

$$\begin{array}{r} 44 \\ \times\ 9 \\ \hline 396 \end{array}$$

8)

$$-7\int_0^1 x(3-x^2)^5\,dx \qquad \text{let } u = 3-x^2$$

$$du = -2x\,dx$$

$$= \frac{7}{2}\int_3^2 u^5\,du$$

$$= -\frac{7}{2}\left[\frac{u^6}{6}\right]_2^3$$

$$= -\frac{7}{2}\left(\frac{3^6}{6} - \frac{2^6}{6}\right)$$

4)

$$\sum_{n=1}^{\infty} (-1)^n (14)^{-n} \le \sum_{n=1}^{\infty} \left(\frac{1}{14}\right)^n$$

converges, geometric series where $|r| < 1$

converges absolutely

at $a = -2$

13) $f(x) = e^{-6x}$ $\qquad e^{12}$

$f'(x) = -\dfrac{e^{-6x}}{6}$ $\qquad -\dfrac{e^{12}}{6}$

$f''(x) = \dfrac{e^{-6x}}{36}$ $\qquad \dfrac{e^{12}}{36}$

$f'''(x) = -\dfrac{e^{-6x}}{216}$ $\qquad -\dfrac{e^{12}}{216}$

$$T_3 = e^{12} - \frac{e^{12}}{6}(x+2) + \frac{e^{12}}{72}(x+2)^2 - \frac{e^{12}}{1296}(x+2)^3$$