Ken mei Test #2 Solutions 12/7/22

Test #2 5:44 pm

Problem 1: Find the Taylor series, centered at $c=6$, for the function $f(x) = \frac{1}{x}$

$f(x) = \sum_{n=0}^{\infty}$ ______ The interval of convergence is: ______

n	$f^{(n)}(a)$	$\frac{f^{(n)}(6)}{n!}$
0	$\frac{1}{x} = x^{-1}$	$\frac{1}{6 \cdot 0!} = \frac{1}{6^1 \cdot 0!}$
1	$-1x^{-2}$	$\frac{-1}{36 \cdot 1!} = \frac{-1}{6^2 \cdot 1!}$
2	$2x^{-3}$	$\frac{2}{216 \cdot 2!} = \frac{2}{6^3 \cdot 2!}$
3	$-6x^{-4}$	$\frac{-6}{1296 \cdot 3!} = \frac{-6}{6^4 \cdot 3!}$
4	$24x^{-5}$	$\frac{24}{7776 \cdot 4!} = \frac{24}{6^5 \cdot 4!}$

$$T(x) = \frac{1}{x} - \frac{1}{36}(x-6) + \frac{2}{216 \cdot 2!}(x-6)^2 - \frac{6}{1296 \cdot 3!}(x-6)^3 + \frac{24}{7776 \cdot 4!}(x-6)^4 + \dots$$

$$\boxed{f(x) = \sum_{n=0}^{\infty} \frac{(-1)^n}{x\, 6^{n+1} \cdot n!}(x-6)^n}$$

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problem 1: Find the Taylor series, centered at $c=6$, for the function $f(x)=\frac{1}{x}$

$$f(x)=\sum_{n=0}^{\infty}\frac{(-1)^n}{x\cdot 6^{n+1}\cdot n!}(x-6)^n$$

Ratio Test $=\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right|<1$ to Test for absolutely convergence

$$a_n=\frac{(-1)^n}{x\,6^{n+1}\cdot n!}(x-6)^n \qquad a_{n+1}=\frac{(-1)^n}{x\,6^{(n+1)+1}\cdot(n+1)!}(x-6)^{n+1}$$

$$\frac{6^{n+1}}{6^{n+2}}=6^{n+1-n-2}=6^{1-2}=6^{-1}=\frac{1}{6}$$

$$\lim_{n\to\infty}\left|\frac{\frac{(-1)^n(x-6)^{n+1}}{x\,6^{(n+2)}\cdot(n+1)!}}{\frac{(-1)^n(x-6)^n}{x\,6^{n+1}\cdot n!}}\right|=\lim_{n\to\infty}\left|\frac{\cancel{(-1)^n}(x-6)^{\cancel{n+1}}}{\cancel{x}6^{n+2}\cdot(n+1)!}\cdot\frac{\cancel{x}\,6^{n+1}\cdot n!}{\cancel{(-1)^n}\cancel{(x-6)^n}}\right|$$

$$\frac{n!}{(n+1)!}=\frac{1}{n+1}$$

$$=\lim_{n\to\infty}\left|\frac{x-6}{6(n+1)}\right|=\lim_{n\to\infty}\frac{|x-6|}{6|n+1|}<1 \qquad R=1 \quad (0-1,0+1) \quad (-1,1)$$

$x=-1$

$$\sum_{n=0}^{\infty}\frac{(-1)^n}{-1\cdot 6^{n+1}\cdot n!}(-1-6)^n=\sum_{n=0}^{\infty}\frac{(-1)^n(-7)^n}{-1\,6^{n+1}\cdot n!}=\sum_{n=0}^{\infty}\frac{(7)^n}{-6^{n+1}\cdot n!}$$

$$=\lim_{n\to\infty}\frac{7^n}{-6^{n+1}\cdot n!}=0$$

$x=1$

$$\sum_{n=0}^{\infty}\frac{(-1)^n}{1\cdot 6^{n+1}\cdot n!}(1-6)^n=\sum_{n=0}^{\infty}\frac{(5)^n}{6^{n+1}\cdot n!} \qquad \lim_{n\to\infty}\frac{5^n}{6^{n+1}\cdot n!}=0$$

Interval of convergence: $(-1,1)$