Ken Mei Test #1 Solutions 12/7/22

Problem 1: Partial Fractions

Evaluate the indefinite integral

$$\int \frac{6x^2 - 6x + 3}{x(x-1)^2}\,dx$$

$$\left(\frac{x(x-1)^2}{1}\right)\frac{6x^2-6x+3}{x(x-1)^2}\,dx = \left(\frac{A}{x} + \frac{B}{(x-1)} + \frac{C}{(x-1)^2}\right) \cdot \frac{x(x-1)^2}{1}$$

$$6x^2 - 6x + 3 = A(x-1)^2 + Bx(x-1) + Cx$$

Let $x = 1$

$$6(1)^2 - 6(1) + 3 = A(1-1)^2 + B(1)(1-1) + C(1)$$

$\boxed{C = 1}$

Let $x = 0$

$$6(0)^2 - 6(0) + 3 = A(0-1)^2 + B(0)(0-1) + C(0)$$

$\boxed{A = 3}$

Let $x = 2$

$$6(2)^2 - 6(2) + 3 = 3(2-1)^2 + B(2)(2-1) + 1(2)$$

$$24 - 12 + 3 = 3 + 2B + 2$$

$$15 = 5 + 2B$$

$$-5 \quad -5$$

$$\frac{10}{2} = \frac{2B}{2}$$

$\boxed{B = 5}$

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Evaluate the indefinite integral

$$\int \frac{6x^2-6x+3}{x(x-1)^2}\,dx = \int \left(\frac{A}{x} + \frac{B}{x-1} + \frac{C}{(x-1)^2} \right) dx$$

$$\int \left(\frac{3}{x} + \frac{5}{x-1} + \frac{1}{(x-1)^2} \right) dx$$

$$\int \frac{3}{x}\,dx = 3 \cdot \int \frac{1}{x}\,dx \quad \begin{matrix} u = x \\ du = dx \end{matrix} \quad \int \frac{1}{u}\,du = \ln|u| + C$$

$$= 3 \cdot \ln|u| + C = \boxed{3\ln|x| + C}$$

$$\int \frac{5}{x-1}\,dx = 5 \cdot \int \frac{1}{x-1}\,dx \quad \begin{matrix} u = x-1 \\ du = 1 \end{matrix} \quad 5 \cdot \int \frac{1}{u}\,du$$

$$= 5 \cdot \ln|u| + C = \boxed{5\ln|x-1| + C}$$

$$\int \frac{1}{(x-1)^2}\,dx \quad \begin{matrix} u = x-1 \\ du = 1 \end{matrix} \quad \int \frac{1}{u^2}\,du = -\frac{1}{u} + C$$

$$= \boxed{\frac{-1}{x-1} + C}$$

$$\boxed{\text{Answer} = 3\ln|x| + 5\ln|x-1| - \frac{1}{x-1} + C}$$