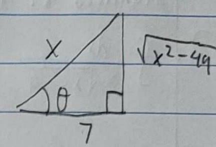
Ken Mei    Group 6 Week #4    12/15/22

Standard #3: trigonometric substitution

1. $\int \frac{\sqrt{x^2-49}}{x^4}\,dx$    $a=7,\ x=7\sec\theta,\ \sec\theta=\frac{x}{7}$

$dx = 7\cdot\sec\theta\cdot\tan\theta\ d\theta$




$= \int \frac{\sqrt{(7\sec\theta)^2-49}}{(7\sec\theta)^4}\cdot 7\sec\theta\tan\theta\ d\theta = \int \frac{\sqrt{49\sec^2\theta-49}}{(7\sec\theta)^4}\cdot 7\sec\theta\tan\theta\ d\theta$

$= \int \frac{\sqrt{(\sec^2\theta-1)\cdot 49}}{(7\sec\theta)^4}\cdot 7\sec\theta\tan\theta\ d\theta = \int \frac{\sqrt{(\tan^2\theta)49}}{(7\sec\theta)^4}\cdot 7\sec\theta\tan\theta\ d\theta$

$\sec\theta = \frac{1}{\cos\theta}$

$\tan\theta = \frac{\sin\theta}{\cos\theta}$

$= \int \frac{7\tan\theta}{(7\sec\theta)^{\cancel{4}\,3}}\cdot \cancel{7\sec\theta}\tan\theta\ d\theta = \int \frac{7\tan^2\theta}{(7\sec\theta)^3}$

$= \int \frac{7\tan^2\theta}{7^3\cdot\sec^3\theta} = \frac{7}{343}\cdot\int\frac{\sin^2\theta}{\cancel{\cos^2\theta}}\cdot\frac{\cos^{\cancel{3}\,1}\theta}{1} = \frac{7}{343}\cdot\int\sin^2\theta\cdot\cos\theta$

u sub:
$u=\sin\theta$
$du=\cos\theta$

$\frac{7}{343}\cdot\int u^2\,du = \frac{7}{343}\cdot\frac{u^3}{3}+C = \frac{7}{343}\cdot\frac{\sin^3\theta}{3}+C$

$\sin\theta = \frac{O}{H} \rightarrow \frac{\sqrt{x^2-49}}{x} = \frac{7}{343}\cdot\frac{\left(\frac{\sqrt{x^2-49}}{x}\right)^3}{\frac{3}{1}} = \frac{7}{343}\cdot\left(\frac{\sqrt{x^2-49}}{x}\right)^3\cdot\frac{1}{3}$

$\boxed{= \frac{7}{343}\cdot\frac{(\sqrt{x^2-49})^3}{3x^3}+C}$

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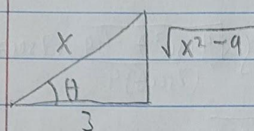
Standard 3: trigonometric substitution

2. $\int \frac{dx}{x\sqrt{x^2-9}}$ $\quad x = 3\sec\theta \quad \sec\theta = \frac{x}{3} \quad dx = 3\sec\theta\tan\theta\, d\theta$

$a = 3 \quad \sec^2\theta - 1 = \tan^2\theta$




$$= \int \frac{3\sec\theta\tan\theta\, d\theta}{3\sec\theta \cdot \sqrt{(3\sec\theta)^2-9}} = \int \frac{3\sec\theta\tan\theta\, d\theta}{3\sec\theta \cdot \sqrt{9\sec^2\theta - 9}}$$

$$= \int \frac{3\sec\theta\tan\theta\, d\theta}{3\sec\theta \cdot \sqrt{9(\sec\theta - 1)}} = \int \frac{\cancel{3\sec\theta}\tan\theta\, d\theta}{\cancel{3\sec\theta}\, 3\tan\theta} = \frac{1}{3}\int \frac{\cancel{\tan\theta}}{\cancel{\tan\theta}}\, d\theta$$

$$= \frac{1}{3}\int 1\, d\theta = \frac{1}{3}\theta + C \qquad \theta = \operatorname{arcsec}\left(\frac{x}{3}\right)$$

$$= \boxed{\frac{1}{3}\operatorname{arcsec}\left(\frac{x}{3}\right) + C}$$

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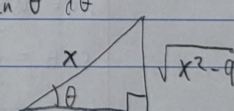
3. $\int \frac{-2}{x^2\sqrt{x^2-9}}\, dx$ $\quad x = 3\sec\theta \quad a = 3 \quad \sec^2\theta - 1 = \tan^2\theta$

$\sec\theta = \frac{x}{3} \quad dx = 3\sec\theta\tan\theta\, d\theta$

$$= -2\int \frac{1 \cdot dx}{x^2\sqrt{x^2-9}} = -2\int \frac{3\sec\theta\tan\theta\, d\theta}{(3\sec\theta)^2\sqrt{(3\sec\theta)^2-9}}$$




$$= -2\int \frac{3\sec\theta\tan\theta\, d\theta}{9\sec^2\theta\sqrt{9(\sec^2\theta - 1)}} = -2 \cdot \frac{1}{3}\int \frac{\sec\theta\tan\theta\, d\theta}{\sec^2\theta\sqrt{9\tan^2\theta}} = \frac{-2}{3}\int \frac{\sec\theta\tan\theta\, d\theta}{\sec^2\theta\, 3\tan\theta}$$

$$= -\frac{2}{3} \cdot \frac{1}{3}\int \frac{\cancel{\sec\theta}\,\cancel{\tan\theta}\, d\theta}{\sec^{\cancel{2}}\theta\, \cancel{\tan\theta}} = -\frac{2}{9}\int \frac{1}{\sec\theta}\, d\theta = \frac{-2}{9}\int \cos(\theta)\, d\theta$$

$$\frac{1}{\sec\theta} = \cos\theta \qquad = \frac{-2}{9} \cdot \sin\theta + C$$

$$\sin\theta = \frac{\sqrt{x^2-9}}{x} \qquad = \boxed{\frac{-2 \cdot \sqrt{x^2-9}}{9x} + C}$$