Ken mei Week #11 Group 6 12/9/22

Part 1: Lesson 17 and 18

Alternating Series test:

Theorem (Alternating Series test): Assume that $\sum_{n=1}^{\infty} a_n$ is a series where

- $a_n \geq 0$ for all n,
- $a_n \geq a_{n+1}$ for all n,
- $\lim_{n \to \infty} a_n = 0$

Then the two alternating series $\sum_{n=1}^{\infty} (-1)^n a_n$ and $\sum_{n=1}^{\infty} (-1)^{n+1} a_n$ both converge.

Absolute and conditional convergence:

Definition: Let $\sum_{n=1}^{\infty} a_n$ be a series

- If $\sum_{n=1}^{\infty} |a_n|$ converges, then $\sum_{n=1}^{\infty} a_n$ converges absolutely
- If $\sum_{n=1}^{\infty} a_n$ converges but $\sum_{n=1}^{\infty} |a_n|$ diverges, then if $\sum_{n=1}^{\infty} a_n$ converges conditionally.

Ratio test:

Theorem (Ratio test): Let $\sum_{n=1}^{\infty} a_n$ be a series with $a_n \neq 0$ for all n.

Define L as follows:

$$L = \lim_{n \to \infty} \left| \frac{a_{n+1}}{a_n} \right|$$

Then:

- If $L < 1$ then the series $\sum_{n=1}^{\infty} a_n$ converges absolvtely,
- If $L > 1$ then the series $\sum_{n=1}^{\infty} a_n$ diverges,
- If $L = 1$, then the ratio test is inconclusive (another test is needed).

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Part 2: Lesson 17 and 18

Root test:

Theorem (Root test): Let $\sum_{n=1}^{\infty} a_n$ be a series with $a_n \neq 0$ for all n.

Define L as follows:

$$L = \lim_{n \to \infty} \sqrt[n]{|a_n|}$$

Then:

- If $L<1$ then the series $\sum_{n=1}^{\infty} a_n$ converges absolutely,
- If $L>1$ then the series $\sum_{n=1}^{\infty} a_n$ diverges,
- If $L=1$, then the ratio test is inconclusive (another test is needed).