Ken Mei Test #2
Cheat sheet
11/23/22

-Geometric: Let $\sum_{n=1}^{\infty} ar^{n-1}$ be a geometric series
- If $|r|<1$ then the series converge to $\frac{a}{1-r}$
- If $|r|\geq 1$ then the series diverges

-P-Series: Let $\sum_{n=1}^{\infty} \frac{1}{n^p}$ be a P-Series where $P>0$
- If $p>1$ then series converges
- If $p\leq 1$ then series diverges

-Integral Test:
If $f(x)$ is positive, decreasing, and continuous function on $[1,\infty)$ such that $f(n)=a_n$ then,
- If $\int_1^{\infty} f(x)\,dx$ converges, then $\sum_{n=1}^{\infty} a_n$ converges
- If $\int_1^{\infty} f(x)\,dx$ diverges, then $\sum_{n=1}^{\infty} a_n$ diverges

- The interval does not need to start at 1
- The function does not necessary always need to be decreasing. It just need to be decreasing for some x-value larger than 1

-Divergence Test: Consider the series $\sum_{n=1}^{\infty} a_n$
- If $\lim_{n\to\infty} a_n \neq 0$ then the series diverges

-Comparison Test: Let $\sum_{n=1}^{\infty} a_n$ and $\sum_{n=1}^{\infty} b_n$ be two series. Assume $0 \leq a_n \leq b_n$ for all n
- Assume that Σa_n diverges. We'll, we knew that $b_n \geq a_n$. So we can conclude that the series Σb_n also diverges.
- Assume that Σb_n converges. We'll, we know that $a_n \leq b_n$. So we can conclude that the series Σa_n also converges.

-Limit Comparison Test: Let a_n and b_n be positive for all natural numbers n. Let $\lim_{n\to\infty} \frac{a_n}{b_n} = L$
- If $L\neq 0$, $L \neq \infty$, the series Σa_n and Σb_n either both converge or they both diverge
- If $L=0$ and if Σb_n converges, then Σa_n also converges
- If $L=\infty$ and if Σb_n diverges, then Σa_n also diverges

-Alternating Series Test: Assume that $\sum_{n=1}^{\infty} a_n$ is a series where
- $a_n \geq 0$ for all n
- $a_n \geq a_{n+1}$ for all n
- $\lim_{n\to\infty} a_n = 0$

Then the two alternating series $\sum_{n=1}^{\infty} (-1)^n a_n$ and $\sum_{n=1}^{\infty} (-1)^{n+1} a_n$ both converge

Absolute and conditional convergence
- If $\sum_{n=1}^{\infty} |a_n|$ converges, then $\sum_{n=1}^{\infty} a_n$ converges absolutely
- If $\sum_{n=1}^{\infty} a_n$ converges but $\sum_{n=1}^{\infty} |a_n|$ diverges, then $\sum_{n=1}^{\infty} a_n$ converges conditionally

-Ratio Test: Let $\sum_{n=1}^{\infty} a_n$ be a series with $a_n \neq 0$ for all n
Define L as follows:

$$L = \lim_{n\to\infty} \left| \frac{a_{n+1}}{a_n} \right|$$

- If $L<1$ then the series Σa_n converges absolutely
- If $L>1$ then the series Σa_n diverges
- If $L=1$ then the ratio test is inconclusive

-Root Test: Let $\sum_{n=1}^{\infty} a_n$ be a series with $a_n \neq 0$ for all n. Define L as follows:

$$L = \lim_{n\to\infty} \sqrt[n]{|a_n|}$$

- If $L<1$ then the series Σa_n converges absolutely
- If $L>1$ then the series Σa_n diverges
- If $L=1$ then the root test is inconclusive

-Taylor Polynomial: The Taylor polynomial of degree N of $f(x)$ centered at $x=a$ is given by:

$$T_N(x) = \sum_{n=0}^{N} \frac{f^{(n)}}{n!}(x-a)^n, \quad T_N(x) = f(a) + f'(a)(x-a) + \frac{f''(a)}{2!}(x-a)^2 + \frac{f'''(a)}{3!}(x-a)^3 + \dots + \frac{f^{(N)}(a)}{N!}(x-a)^N$$

-Power Series:

$$\sum_{n=0}^{\infty} a_n (x-c)^n = a_0 + a_1(x-c) + a_2(x-c)^2 + \dots + a_n(x-c)^n + \dots$$

- If $R=0$, the power series only converges at c
- If $R=\infty$, the power series converges for all values of x
- The value of R tells the series will converge on the interval $(c-R, c+R)$

Ex: $\sum_{n=1}^{\infty} (-1)^n n x^n$ Using Ratio Test

$$\lim_{n\to\infty} \left| \frac{(n+1)x^{(n+1)}}{n x^n} \right| = \lim_{n\to\infty} \left| \frac{(n+1)x}{n} \right| = |x| < 1$$

$R=1$ $(0-1, 0+1)$

$x=-1$

$$\sum_{n=1}^{\infty} (-1)^n n(-1)^n = \sum_{n=1}^{\infty} n \qquad \lim_{n\to\infty} n \neq 0$$

series diverges at $x=-1$ by the nth term divergence test

$x=1$

$$\sum_{n=1}^{\infty} (-1)^n n(1)^n \qquad \lim_{n\to\infty} n \neq 0$$

series also diverge at $x=1$ by the nth term divergence test

Interval of convergence $= (-1, 1)$

-Taylor Series:

$$T(x) = \sum_{n=0}^{\infty} \frac{f^{(n)}(a)}{n!}(x-a)^n = f(a) + f'(a)(x-a) + \frac{f''(a)}{2}(x-a)^2 + \frac{f'''(a)}{3!}(x-a)^3 + \dots$$

Find the Taylor series for $f(x) = \ln(x)$ at $a=2$

$f(x) = \ln(x)$, $f'(x) = x^{-1}$, $f''(x) = -1x^{-2}$, $f'''(x) = 2x^{-3}$, $f^4(x) = -6x^{-4}$

$f(2) = \ln(2)$, $f'(2) = \frac{1}{2}$, $f''(2) = \frac{-1}{2 \cdot 2^2}$, $f'''(2) = \frac{1}{3 \cdot 2^3}$, $f^4(2) = \frac{-1}{4 \cdot 2^4}$

$$T(x) = \ln(2) + \frac{1}{2}(x-2) - \frac{1}{2 \cdot 2^2}(x-2)^2 + \frac{1}{3 \cdot 2^3}(x-2)^3 - \frac{1}{4 \cdot 2^4}(x-2)^4$$

$$= \ln(2) + \sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n \cdot 2^n}(x-2)^n$$

$\frac{d}{dx} e^x = e^x$, $\frac{d}{dx}\ln(x) = \frac{1}{x}$

$\frac{d}{dx}\sin(x) = \cos(x)$, $\frac{d}{dx}\cos(x) = -\sin(x)$

$\frac{d}{dx} -\cos(x) = \sin(x)$, $\frac{d}{dx} -\sin(x) = -\cos(x)$, $\frac{d}{dx}\tan(x) = \sec^2(x)$

$\frac{d}{dx}\sec(x) = \sec(x)\tan(x)$, $\frac{d}{dx}\cot(x) = -\csc^2(x)$, $\frac{d}{dx}\csc(x) = -\cot(x)\cdot\csc(x)$

$\frac{1}{\cos^2(x)} = \sec^2(x)$, $\frac{1}{\sin^2(x)} = \csc^2(x)$, $\frac{1}{\tan^2(x)} = \cot^2(x)$

$\int \sin(x)dx = -\cos(x) + C$

$\int \cos(x)\,dx = \sin(x) + C$

$\int \tan(x)dx = -\ln|\cos(x)| + C$

$\int \csc(x)\,dx = \ln\left|\tan\left(\frac{x}{2}\right)\right| + C$

$\int \cot(x)\,dx = \ln|\sin(x)| + C$

$\int \sec^2(x)\,dx = \tan(x) + C$

$\int \tan(x)\sec(x)\,dx = \sec(x) + C$

$\int \csc^2(x)\,dx = -\cot(x) + C$