$\frac{d}{dx} e^x = e^x$

$\frac{d}{dx} \sin(x) = \cos(x)$

$\frac{d}{dx} \cos(x) = -\sin(x)$

$\frac{d}{dx} \tan(x) = \sec^2(x)$

$\frac{d}{dx} \sec(x) = \sec(x)\tan(x)$

$\frac{d}{dx} \cot(x) = -\csc^2(x)$

$\frac{d}{dx} \ln(x) = \frac{1}{x}$

$\frac{d}{dx} \csc = -\cot(x)\csc(x)$

$\frac{1}{\cos^2(x)} = \sec^2(x)$

$\frac{1}{\sin^2(x)} = \csc^2(x)$

$\frac{1}{\tan^2(x)} = \cot^2(x)$

$\int \sin(x)\,dx = -\cos(x) + C$

$\int \cos(x)\,dx = \sin(x) + C$

$\int \tan(x)\,dx = -\ln|\cos(x)| + C$

$\int \csc(x)\,dx = \ln\left|\tan\left(\frac{x}{2}\right)\right| + C$

$\int \cot(x)\,dx = \ln|\sin(x)| + C$

$\int \sec^2(x)\,dx = \tan(x) + C$

$\int \tan(x)\sec(x)\,dx = \sec(x) + C$ $\quad \int \csc^2(u)\,du = -\cot(u) + C$

Type 1:

- $\int_a^{\infty} f(x)\,dx = \lim_{b \to \infty} \int_a^b f(x)\,dx$
- $\int_{-\infty}^{b} f(x)\,dx = \lim_{a \to -\infty} \int_a^b f(x)\,dx$
- $\int_{-\infty}^{\infty} f(x)\,dx = \int_{-\infty}^{b} f(x)\,dx + \int_b^{\infty} f(x)\,dx$

Ken Mei

Test #1 cheat sheet:

Pythagorean Indentity

- $1 - \sin^2\theta = \cos^2\theta$
- $1 + \tan^2\theta = \sec^2\theta$
- $\sec^2\theta - 1 = \tan^2\theta$

$\sqrt{a^2 - x^2}$ let $x = a\sin\theta$

$\sqrt{a^2 - x^2} = a\cos\theta$

$\sqrt{a^2 + x^2}$ let $x = a\tan\theta$

$\sqrt{a^2 + x^2} = a\sec\theta$

$\sqrt{x^2 - a^2}$ let $x = a\sec\theta$

$\sqrt{x^2 - a^2} = \pm a\tan\theta$

-Standard #1 U-substitution:

Evaluate: $\int e^{3x} \sin(e^{3x})\,dx$

$u = e^{3x}$ $\quad \frac{du}{3} = \frac{\cancel{3}e^{3x}}{\cancel{3}}\,dx \to \frac{1}{3}du = e^{3x}dx$

$= \int \sin(e^{3x})\, e^{3x}\,dx \to \frac{1}{3} \cdot \int \sin(u)\,du$

$= \frac{1}{3} \cdot (-\cos(u)) + C = \boxed{-\frac{1}{3}\cos(e^{3x}) + C}$

-Standard #2 integration by parts

$\int x^2 \sin(x)\,dx$ $\qquad uv - \int v\,du$

$u = x^2 \to du = 2x\,dx$

$dv = \sin(x)\,dx \to v = -\cos(x)$

$\int x^2 \sin(x)\,dx = x^2 \cdot -\cos(x) - \int -\cos(x) \cdot 2x\,dx$

$= x^2 \cdot -\cos(x) + 2\int \cos(x)\,x\,dx$

Integration by parts twice

$= \int \cos(x)\,x\,dx$

$u = x \to du = dx$

$dv = \cos(x)\,dx \to v = \sin(x)$

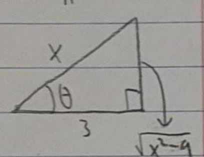
$= x \cdot \sin(x) - \int \sin(x)\,dx$

$= x\sin(x) - (-\cos(x)) + C$

$= \boxed{-x^2\cos(x) + 2 \cdot (x\sin(x) + \cos(x)) + C}$

-Standard #3 Trigonometric substitution

$\sec = \frac{H}{A}$



$\int \frac{dx}{x\sqrt{x^2-9}}$ $\quad x = 3\sec(\theta)$ $\quad a = 3$ $\quad \sec\theta = \frac{x}{3}$

$dx = 3\sec\theta \cdot \tan\theta\,d\theta$

$= \int \frac{3\sec\theta \cdot \tan\theta\,d\theta}{3\sec\theta \cdot \sqrt{(3\sec\theta)^2 - 9}} = \int \frac{3\sec\theta\tan\theta\,d\theta}{3\sec\theta \cdot \sqrt{9\sec^2\theta - 9}}$

$\sec^2\theta - 1 = \tan^2\theta$

$= \int \frac{\cancel{3\sec\theta} \cdot \tan\theta\,d\theta}{\cancel{3\sec\theta} \cdot \sqrt{9(\sec^2\theta - 1)}} = \int \frac{\tan\theta\,d\theta}{3\tan\theta} = \frac{1}{3}\int \frac{\cancel{\tan\theta}\,d\theta}{\cancel{\tan\theta}} = \frac{1}{3}\int 1\,d\theta = \frac{1}{3}\theta + C$

$\sec\theta = \frac{x}{3}$ $\quad \sec\theta = \operatorname{arcsec}\left(\frac{x}{3}\right)$ $\qquad \frac{1}{3}\theta + C = \boxed{\frac{1}{3}\operatorname{arcsec}\left(\frac{x}{3}\right) + C}$

$\sin\theta = \frac{O}{H}$ $\quad \cot\theta = \frac{A}{O}$

$\cos\theta = \frac{A}{H}$ $\quad \csc\theta = \frac{H}{O}$

$\tan\theta = \frac{O}{A}$ $\quad \cot\theta = \frac{\cos\theta}{\sin\theta}$

$\sec\theta = \frac{H}{A}$ $\quad \tan\theta = \frac{\sin\theta}{\cos\theta}$

$\sec\theta = \frac{1}{\cos\theta}$

Improper Integration:

Type 2:

$\int_a^b f(x)\,dx = \lim_{t \to a^+} \int_t^b f(x)\,dx$

$\int_a^b f(x)\,dx = \lim_{t \to b^-} \int_a^t f(x)\,dx$

$\int_a^b f(x)\,dx = \int_a^c f(x)\,dx + \int_c^b f(x)\,dx$

Standard #4 Partial Fraction decomposition

$\int \frac{3x+9}{x^2-4x-5}\,dx \to \frac{3x+9}{(x-5)(x+1)} = \frac{A}{(x-5)} + \frac{B}{(x+1)}$ $\qquad \cancel{(x-5)(x+1)} \cdot \frac{3x+9}{\cancel{(x-5)(x+1)}} = \left(\frac{A}{(x-5)} + \frac{B}{(x+1)}\right) \cdot (x-5)(x+1)$

$3x + 9 = A(x+1) + B(x-5)$

Let $x = -1 \to 3(-1) + 9 = A(-1+1) + B(-1-5)$

$\frac{6}{-6} = \frac{\cancel{-6}B}{\cancel{-6}}$ $\quad \boxed{B = -1}$

Let $x = 5 \to 3(5) + 9 = A(5+1) + B(5-5)$

$\frac{24}{6} = \frac{\cancel{6}A}{\cancel{6}}$ $\quad \boxed{A = 4}$

$= \int \left(\frac{4}{x-5} - \frac{1}{x+1}\right)dx$

$= 4\int \frac{1}{x-5}\,dx - \int \frac{1}{x+1}\,dx$

$= \boxed{4\ln|x-5| - \ln|x+1| + C}$

$\int \frac{1}{u}\,du = \ln|u| + C$

$\int \frac{1}{x-5}\,dx$ $\quad u = x-5$, $du = dx$

$\int \frac{1}{u}\,du = \ln|x-5| + C$

Note: Quadratic factors: Each distinct factor in the form (ax^2+bx+c) must include a term in the following form $\frac{Bx+C}{ax^2+bx+c}$. For each repeated factor, $(ax^2+bx+c)^n$, we must include n terms of the form $\frac{Bx+C}{ax^2+bx+c} + \frac{Dx+E}{(ax^2+bx+c)^2}$

Standard #5 improper Integrals

$\int_7^{\infty} \frac{1}{x^{\frac{6}{5}}}\,dx = \lim_{b \to \infty} \int_7^b \frac{1}{x^{\frac{6}{5}}}\,dx = \lim_{b \to \infty} \int_7^b x^{-\frac{6}{5}} = \lim_{b \to \infty} \left. \frac{x^{-\frac{1}{5}}}{-\frac{1}{5}} \right]_7^b = \lim_{b \to \infty} \left. \frac{-5}{x^{\frac{1}{5}}} \right]_7^b = \lim_{b \to \infty} \left(\frac{-5}{(b)^{\frac{1}{5}}} - \left(\frac{-5}{(7)^{\frac{1}{5}}}\right)\right)$

power rule $\frac{x^{n+1}}{n+1}$

$\downarrow$
$0 + \frac{5}{7^{\frac{1}{5}}}$

Rough work:

As b approaches ∞

$b^{\frac{1}{5}} = \sqrt[5]{b} \to 0$

$\boxed{\text{Answer} = \frac{5}{7^{\frac{1}{5}}} \text{ converges}}$