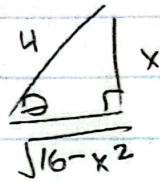


Group 4

Standard #3

$$1) \int \sqrt{16-x^2} dx$$
$$= 4 \int \sqrt{16(1-\sin^2 \theta)} \cdot \cos \theta d\theta$$



$$\text{let } x = 4\sin \theta$$

$$dx = 4\cos \theta d\theta$$

$$\cos^2 \theta = 1 - \sin^2 \theta$$

$$= 4 \int 4\cos \theta \cdot \cos \theta d\theta$$

$$= 16 \int \cos^2 \theta d\theta$$

$$\cos^2 \theta = \frac{\cos 2\theta + 1}{2}$$

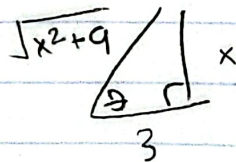
$$= 8 \int [\cos 2\theta + 1] d\theta$$

$$= 4\sin 2\theta + 8\theta + C$$

$$\sin 2\theta = 2\sin \theta \cos \theta$$

$$= \frac{x\sqrt{16-x^2}}{2} + 8\sin^{-1}\left(\frac{x}{4}\right) + C$$

$$2) \int \frac{-8x^3}{\sqrt{x^2+9}} dx$$



$$\text{let } x = 3\tan \theta$$

$$dx = 3\sec^2 \theta d\theta$$

$$= -8 \int \frac{27\tan^3 \theta}{\sqrt{9(\tan^2 \theta + 1)}} \cdot 3\sec^2 \theta d\theta$$

$$\sec^2 \theta = \tan^2 \theta + 1$$

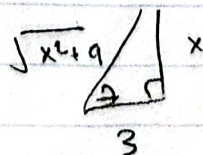
$$= -8 \int 27\tan^3 \theta \sec \theta d\theta$$

$$= -8(27) \int (\sec^2 \theta - 1) \tan \theta \sec \theta d\theta$$

$$\text{let } u = \sec \theta$$

$$du = \tan \theta \sec \theta d\theta$$

$$= -216 \int (u^2 - 1) du$$



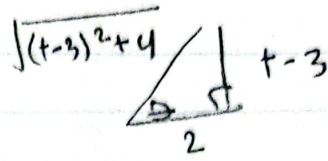
$$= -72\sec^3 \theta + 216\sec \theta + C$$

$$= -\frac{8}{3}(x^2+9)^{\frac{3}{2}} + 72\sqrt{x^2+9} + C$$

3)

$$\int \frac{10}{\sqrt{t^2 - 6t + 13}} dt$$

$$= 10 \int \frac{1}{\sqrt{(t-3)^2 + 4}} dt$$



$$\text{let } t-3 = 2 \tan \theta$$

$$dt = 2 \sec^2 \theta d\theta$$

$$= 10 \int \frac{1}{\sqrt{4(\tan^2 \theta + 1)}} \cdot 2 \sec^2 \theta d\theta$$

$$= 10 \int \sec \theta d\theta$$

$$= 10 \ln |\sec \theta + \tan \theta| + C$$

$$= 10 \ln \left| \frac{\sqrt{(t-3)^2 + 4} + (t-3)}{2} \right| + C$$