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Test 7

1) $\int \frac{\sqrt{x^2-4}}{x^6}\,dx$ $\qquad x = 2\sec\theta$

$\qquad dx = 2\sec\theta\tan\theta\,d\theta$

$= \int \frac{2\tan^2\theta\,(2\sec\theta\tan\theta)}{64\sec^6\theta}\,d\theta$

$u = x^2-4$

$du = 2x\,dx$

$= \int \frac{\tan^3\theta}{16\sec^5\theta}\,d\theta$

$= \frac{1}{16}\int \frac{\tan^2\theta\tan\theta}{\sec^2\theta\sec^2\theta\sec\theta}\,d\theta$

2) $\int \frac{\ln x}{x^6}\, dx = \int \ln x (x^{-6})\, dx$

$u = \ln x \qquad du = x^{-1} dx$

$dv = x^{-6} dx \qquad v = -\frac{x^{-5}}{5} dx$

$uv - \int v\,du$

$\ln x(-x^5/5) - \int -x^{-5}/5 \; x^{-1} dx$

$\cancel{u = x^{-5}}$

$\ln x(-x^5/5) + \frac{1}{5}\int 1/x^5 \; x^{-1} dx$

$\ln x(-\frac{x^5}{5}) + \frac{1}{5}\int x^{-6} dx$

$\ln x(-\frac{x^5}{5}) + \frac{1}{5}(-\frac{x^5}{5}) + C$

$-\frac{x^5 \ln x}{5} + \left(-\frac{x^5}{25}\right) + C$

$-\frac{x^5 \ln x}{5} - \frac{x}{5} + C$

3) $$\int_{-\infty}^{-1} e^{8t}\,dt = \lim_{a\to-\infty} \int_{a}^{-1} e^{8t}\,dt$$

$$= \lim_{a\to-\infty} \left[e^{8t} \right]_{a}^{-1}$$

$$= \lim_{a\to-\infty} \left[e^{8(-1)} - e^{8(a)} \right]$$

4) $f(x) = \frac{6}{x^3} - \frac{7}{x^7}$

$F(x) = f'(x)$

$F(1) = 0$

$$f'(x) = \left(\frac{0(x^3) - 6(3x^2)}{(x^3)^2} \right) - \left(\frac{0(7x^6) - (7)(x^7)}{(x^7)^2} \right)$$

$$= \frac{18x^2}{x^6} - \frac{7x^7}{x^{14}}$$

$f(x) = \frac{18}{x^4} - \frac{7}{x^7}$ $\qquad 0 = \frac{18}{1^4} - \frac{7}{1^7} = 11$

$$0 = \left(\frac{0(1^3) - 6(3(1)^2)}{(1^3)^2} \right) - \left(\frac{0(7(1)^6) - 7(1^7)}{(1^7)^2} \right)$$

$$0 = \left(\frac{0-18}{1} \right) - \left(\frac{0-7}{1} \right)$$

$= -18 + 7 =$

5) $g(r) = \int_0^r \sqrt{x^2+1}\ dx$

$$= \int_0^r (x^2+1)^{1/2}\ dx = \left[\frac{(x^2+1)^{1/2+1}}{1/2+1}\right]_0^r$$

$$= \left[\frac{(x^2+1)^{3/2}}{3/2}\right]_0^r = \left[\frac{2(x^2+1)^{3/2}}{3}\right]_0^r$$

$$= \frac{2(r^2+1)^{3/2}}{3} - \frac{2(0^2+1)^{3/2}}{3}$$

$$= \frac{2(r^2+1)^{3/2}}{3} - \frac{\sqrt{8}}{3}$$

$$g(r) = \frac{\sqrt{(2r^2+2)^3}}{3} - \frac{\sqrt{8}}{3}$$

$$g'(r) = \frac{27\sqrt{6r^2+6}}{2} - \frac{9}{2\sqrt{8}}$$

$$\frac{3}{2}(2r^2+2)^{1/2}$$

$$\frac{\frac{\sqrt{6r^2+6}}{2}(3)}{9} \qquad \frac{3\sqrt{6r^2+6}}{2} \cdot \frac{9}{1} \qquad \frac{27\sqrt{6r^2+6}}{2}$$

$$\frac{\frac{1}{2}(8)^{(-\frac{1}{2})}}{9} \qquad \frac{1}{2} \cdot \frac{1}{\sqrt{8}} \qquad \frac{1}{2\sqrt{8}} \cdot \frac{9}{1} = \frac{9}{2\sqrt{8}}$$

7) $\int \frac{\ln^5(z)}{z}\,dz$

$u = \ln(z)$ $\qquad = \int u^5\,du = \frac{u^6}{6} + C$

$du = \frac{1}{z}\,dz$

$= \boxed{\frac{\ln^6(z)}{6} + C}$

$$8)\ \int \frac{4x^2-18x+27}{x(x-3)^2}\,dx = \int \frac{A}{x}\,dx + \int \frac{B}{x-3}\,dx + \int \frac{C}{(x-3)^2}\,dx$$

$$\frac{4x^2-18x+27}{x(x-3)^2} = \frac{A}{x} + \frac{B}{x-3} + \frac{C}{(x-3)^2}$$

$$4x^2-18x+27 = A(x-3)^2 + B(x)(x-3) + C(x)$$

$x=0$, $4(0)^2-18(0)+27 = A(0-3)^2 + B(0)(0-3) + C(0)$

$$27 = A(-3)^2 + B(0)(-3) + C(0)$$
$$27 = A(9)$$
$$3 = A$$

$x=3$, $4(3)^2-18(3)+27 = A(3-3)^2 + B(3)(3-3) + C(3)$

$$9 = C(3)$$
$$3 = C$$

$x=2$, $4(2)^2-18(2)+27 = 3(2-3)^2 + B(2)(2-3) + 3(2)$

$$7 = 3 + B(-2) + 6$$
$$7 = 9 + B(-2)$$
$$-2 = B(-2)$$
$$1 = B$$

$$\int \frac{3}{x}\,dx + \int \frac{1}{x-3}\,dx + \int \frac{3}{(x-3)^2}\,dx$$

$$3\ \frac{(x-3)^{-2+1}}{-2+1} \qquad \frac{(x-3)^{-1}}{-1} \qquad -(x-3)^{-1} \qquad 3 \cdot -\frac{1}{(x-3)}$$

$$= \boxed{3\ln|x| + \ln|x-3| - \frac{3}{x-3} + C}$$