SOLUTIONS

MAT 1575 **Name (Print):** ______________
Fall 2014
Professor K. Poirier
Test #1
October 2 **Time Limit: 100 Minutes**

This exam contains 6 pages and 10 problems. Check to see if any pages are missing. Print your name on the top of this page, and put your initials on the top of every page, in case the pages become separated.

You may use a calculator on this test. No other aids are allowed. Show all your work.

Total: 50 points, including 2 points for overall style (this includes using all notation correctly and presenting the work logically).

CHECK ALL ANTIDERIVATIVES BY DIFFERENTIATING.

ONE POINT FOR EACH OF THE RELEVANT QUESTIONS IS FOR THIS CHECK

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1. (4 points) Evaluate the integral. Leave your answer in simplest form.

$$\int_0^5 dx = x\Big]_0^5 = 5-0 = 5$$

$$\frac{d}{dx}(x) = 1$$

2. (4 points) Differentiate. SECOND FUNDAMENTAL THEOREM.

$$\frac{d}{dx}\int_2^x \sin(t)du = \sin(x)$$

3. (5 points) Evaluate the integral.

SUBSTITUTION

$$\int \frac{3x^2}{\sqrt[5]{x^3+4}}dx$$

let $u = x^3+4$

$du = 3x^2 dx$

$$= \int \frac{du}{u^{\frac{1}{5}}}$$

$$= \frac{5}{4} u^{\frac{4}{5}} + c$$

$$= \frac{5}{4}(x^3+4)^{\frac{4}{5}} + c$$

$$= \frac{5}{4}\sqrt[5]{x^3+4}^{\,5} + c$$

CHECK

$$\frac{d}{dx}\ \frac{5}{4}(x^3+4)^{\frac{4}{5}}$$

$$= \cancel{\frac{4}{5}} \cdot \cancel{\frac{5}{4}} (x^3+4)^{-\frac{1}{5}} \cdot 3x^2$$

$$= \frac{3x^2}{\sqrt[5]{x^3+4}}$$

✓

4. (5 points) Evaluate the integral. Leave your answer in simplest form. Do not approximate by a decimal.

SUBSTITUTION

$$\int_1^{\sqrt{3}} \frac{dx}{(\tan^{-1}(x))(1+x^2)}$$

let $u = \tan^{-1}(x)$

$du = \frac{1}{1+x^2}dx$

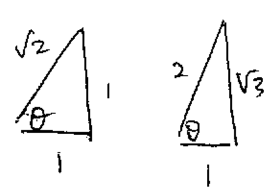
CHECK

$$\frac{d}{dx}\ln(\tan^{-1}(x))$$

$$= \frac{1}{\tan^{-1}(x)} \cdot \frac{1}{1+x^2}$$

✓

$$= \int_{x=1}^{x=\sqrt{3}} \frac{1}{u}\,du$$



$$= \cancel{\int_{x=1}^{x=\sqrt{3}}} \ln|u| \Big]_{x=1}^{x=\sqrt{3}}$$

$$= \ln|\tan^{-1}(x)| \Big]_{x=1}^{x=\sqrt{3}}$$

$$= \ln(\tan^{-1}(x)) \Big]_{x=1}^{x=\sqrt{3}}$$

since $\tan^{-1}(x)$ is >0 between 1 and $\sqrt{3}$

$$= \ln(\tan^{-1}(\sqrt{3})) - \ln(\tan^{-1}(1))$$

$$= \ln\left(\frac{\pi}{3}\right) - \ln\left(\frac{\pi}{4}\right) = \ln\left(\frac{\frac{\pi}{3}}{\frac{\pi}{4}}\right) = \ln\left(\frac{4}{3}\right)$$

5. (5 points) Evaluate the integral.

$$\int \frac{dx}{x^2\sqrt{x^2-25}}$$

TRIG SUBSTITUTION: let $x = 5\sec\theta$

so $dx = 5\sec\theta\tan\theta\,d\theta$

and $x^2 = 25\sec^2\theta$

so $x^2 - 25 = 25\sec^2\theta - 25$

$= 25(\sec^2\theta - 1)$

$= 25\tan^2\theta$

and $\sqrt{x^2-25} = \sqrt{25\tan^2\theta}$

$= 5\tan\theta$

$$= \int \frac{\cancel{5\sec\theta}\,\cancel{\tan\theta}\,d\theta}{25\sec^{\cancel{2}}\theta \cdot \cancel{5\tan\theta}}$$

$$= \int \frac{d\theta}{25\sec\theta}$$

$$= \frac{1}{25}\int \cos\theta\,d\theta$$

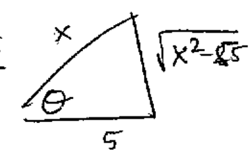
$$= \frac{1}{25}\sin\theta + C$$

$$= \frac{1}{25} \cdot \frac{\sqrt{x^2-25}}{x} + C$$

since $x = 5\sec\theta$

$\frac{x}{5} = \sec\theta = \frac{1}{\cos\theta}$

So $\cos\theta = \frac{5}{x} = \frac{A}{H}$



and $\sin\theta = \frac{O}{H} = \frac{\sqrt{x^2-25}}{x}$

CHECK: $\frac{d}{dx}\left(\frac{\sqrt{x^2-5}}{25x} + C\right) = \frac{\left(\frac{1}{\cancel{2}}(x^2-25)^{-\frac{1}{2}} \cdot \cancel{2}x\right)(25x) - (x^2-25)^{\frac{1}{2}} \cdot 25}{25^2x^2}$

$$= \frac{25(x^2-25)^{-\frac{1}{2}}\left[x^2 - (x^2-25)\right]}{25^2x^2}$$

$$= \frac{\cancel{25}}{\cancel{25}\sqrt{x^2-5} \cdot x^2}$$

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6. (5 points) Evaluate the integral.

$$\int \frac{dx}{x\ln(x)}$$

SUBSTITUTION.

let $u = \ln(x)$

so $du = \frac{1}{x}dx$

$$= \int \frac{1}{u}\,du$$

$$= \ln|u| + C$$

$$= \ln|\ln(x)| + C.$$

CHECK:

(1) If $\ln(x) \geq 0$, then

$$\frac{d}{dx}\left(\ln(\ln(x))\right) = \frac{1}{\ln(x)} \cdot \frac{1}{x} \checkmark$$

(2) If $\ln(x) < 0$, then

$$\frac{d}{dx}\left(\ln(-\ln(x))\right) = \frac{1}{-\ln(x)}\left(-\frac{1}{x}\right)$$

$$= \frac{1}{x\ln(x)} \checkmark.$$

7. (5 points) Evaluate the integral.

$$\int \sin^5(x)dx = \int \cancel{\sin^3(x)\sin^2(x)}$$

$$= \int \sin^4(x)\sin(x)\,dx$$

$$= \int (\sin^2(x))^2 \sin(x)\,dx.$$

$$= \int (1-\cos^2(x))^2 \sin(x)\,dx.$$

$$= -\int (1-u^2)^2\,du$$

$$= -\int (1-2u^2+u^4)\,du$$

$$= -u + \frac{2}{3}u^3 - \frac{1}{5}u^5 + C.$$

$$= -\cos(x) + \frac{2}{3}\cos^3(x) - \frac{1}{5}\cos^5(x) + C.$$

SUBSTITUTION.

let $\cos(x) = u$

~~so $\cos(x)dx = du$~~

so $du = -\sin(x)dx$

CHECK - separate page

$$\int uv' = uv - \int u'v.$$

8. (5 points) Evaluate the integral. INTEGRATION BY PARTS

$$\int \underset{u}{x} \underset{v'}{\cos(x)dx} = x\sin(x) - \int \sin(x)\,dx$$

$$= x\sin(x) + \cos(x) \cancel{dx} + C$$

$u = x \quad v = \sin(x)$

$u' = 1 \quad v' = \cos(x)$

CHECK. $\cancel{\frac{d}{dx}(x\cos(x) + C)}$

$$\frac{d}{dx}\left(x\sin(x) + \cos(x) + C\right)$$

$$= \sin(x) + x\cos(x) - \sin(x)$$

$$= x\cos(x) \quad ✓$$

9. (5 points) Evaluate the integral. INTEGRATION BY PARTS TWICE

$$\int \underset{u}{e^x} \underset{v'}{\cos(x)dx}$$

$u = e^x \quad v = \sin(x)$

$u' = e^x \quad v' = \cos(x)$

$$= e^x\sin(x) - \int e^x\sin(x)\,dx$$

$$= e^x\sin(x) - \left(-e^x\cos(x) + \int e^x\cos(x)dx\right)$$

$u = e^x \quad v = -\cos(x)$

$u' = e^x \quad v' = \sin(x)$

So

$$2\int e^x\cos(x)dx = e^x\sin(x) + e^x\cos(x) + C$$

and

$$\int e^x\cos(x)dx = \frac{e^x}{2}\left(\sin(x) + \cos(x)\right) + C$$

CHECK. $\frac{d}{dx}\left(\frac{e^x}{2}(\sin(x) + \cos(x)) + C\right) = \frac{e^x}{2}(\sin(x) + \cos(x)) + \frac{e^x}{2}(\cos(x) \cancel{-\sin(x)})$

$$= \cancel{2}\frac{e^x}{\cancel{2}}\cos(x) \quad ✓$$

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3. The total number of downloads of a new video game was 1.5 million in 2012 and 2.5 million in 2013. Assume that the number of downloads grows exponentially.

(a) (4 points) Determine a formula for the total number of downloads after t years.

$f(x) =$

$C b^{2012} = 1.5m$

$C b^{2013} = 2.5m$

$\Rightarrow b = \frac{2.5}{1.5} = 1.667$

$C \cdot 1.667^{2012} = 1.5m$

$C = \frac{1.5m}{1.667^{2012}}$

(b) (4 points) Determine how long it takes for the total number of downloads to reach 15 million.

$$\int \sin^5(x) = \int \sin^4(x)\sin(x)\,dx$$

$$= \int (1-\cos^2(x))^2 \sin(x)\,dx$$

let $u = \cos(x)$
$du = -\sin(x)dx$

$$= -\int (1-u^2)^2\,du$$

$$= -\int (1-2u^2+u^4)\,du$$

$$= -\left(u - \tfrac{2}{3}u^3 + \tfrac{1}{5}u^5\right) + C$$

$$\frac{d}{du}\left(u - \tfrac{2}{3}u^3 + \tfrac{1}{5}u^5\right) = 1 - 2u^2 + u^4.$$

$$= -\left(\cos(x) - \tfrac{2}{3}\cos^3(x) + \tfrac{1}{5}\cos^5(x)\right) + C$$

$$\frac{d}{dx}\left(-\cos(x) + \tfrac{2}{3}\cos^3(x) - \tfrac{1}{5}\cos^5(x)\right)$$

$$= \sin(x) - 2\cos^2(x)\sin(x) + \cos^4(x)\sin(x)$$

$$= \sin(x)\left(1 - 2\cos^2(x) + \cos^4(x)\right).$$

$$= \sin(x)\left(1 - \cos^2(x) - \cos^2(x) + \cos^4(x)\right).$$

$$= \sin(x)\left(\sin^2(x) - \cos^2(x)\left(1 - \cos^2(x)\right)\right)$$

$$= \sin(x)\left(\sin^2(x) - \underbrace{\cancel{(1-\cos^2(x))}}_{\cos^2(x)}\underbrace{\cancel{(1-\cos^2(x))}}_{\sin^2(x)}\right).$$

$$= \sin(x)\left(\sin^2(x)\left(1-\cos^2(x)\right)\right).$$

$$= \sin(x)\left(\sin^2(x)\sin^2(x)\right).$$

✓ -