MAT 1475H Name (Print): DRAFT
Spring 2014
Professor K. Poirier
Test #2
May 12, 2014 **Time Limit: 100 Minutes**

This exam contains 7 pages and 8 problems. Check to see if any pages are missing. Print your name on the top of this page, and put your initials on the top of every page, in case the pages become separated.

You may use a calculator on this test. No other aids are allowed.

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1. Find $\frac{dy}{dx}$.

(a) (5 points)

$$x^2 + y^2 = 25$$

$$2x + 2y\frac{dy}{dx} = 0$$

$$2y\frac{dy}{dx} = -2x$$

$$\frac{dy}{dx} = \frac{-2x}{2y}$$

$$\frac{dy}{dx} = -\frac{x}{y}$$

(b) (5 points)

$$xe^y = 2xy + y^3$$

$$e^y + xe^y\frac{dy}{dx} = 2y + 2x\frac{dy}{dx} + 3y^2\frac{dy}{dx}$$

$$(xe^y - 2x - 3y^2)\frac{dy}{dx} = 2y - e^y$$

$$\frac{dy}{dx} = \frac{2y - e^y}{xe^y - 2x - 3y^2}$$

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2. (5 points) Evaluate the limit:

$$\lim_{x\to 0}\left(\frac{1}{\sin(x)}-\frac{1}{x}\right) = \lim_{x\to 0} \frac{x-\sin(x)}{x\sin(x)} \quad \frac{0}{0}$$

$$= \lim_{x\to 0} \frac{1-\cos(x)}{\sin(x)+x\cos(x)} \quad \text{L'Hôpital's rule.}$$

$$= \lim_{x\to 0} \frac{\sin(x)}{\cos(x)+\cos(x)-x\sin(x)} \quad \text{L'Hôp}$$

$$= \frac{0}{1+1-0}$$

$$= \frac{0}{2}$$

$$= 0$$

3. Evaluate the indefinite integrals.

(a) (5 points)

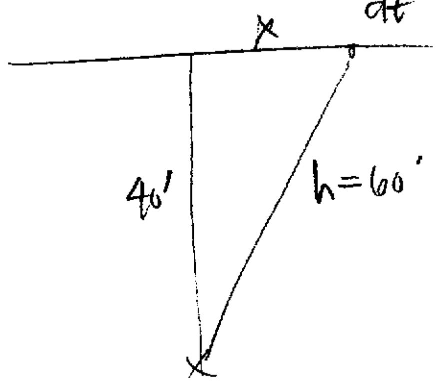
$$\int (x^2+x+1)dx = \frac{1}{3}x^3 + \frac{1}{2}x^2 + 1 + c$$

(b) (5 points)

$$\int \frac{x\sin(x)+xe^x+1}{x}dx = \int\left(\sin(x)+e^x+\frac{1}{x}\right)dx$$

$$= -\cos(x) + e^x + \ln(|x|) + c$$

$\frac{dx}{dt} = ?$



$\frac{dh}{dt} = 50$ mph

(4)

$$h^2 = x^2 + 40^2$$

$$2h\frac{dh}{dt} = 2x\frac{dx}{dt}$$

50

$\frac{60}{5280}$

$$\frac{h \frac{dh}{dt}}{x} = \frac{dx}{dt}$$

$\frac{44}{5280}$

$$\frac{60 \cdot 50}{44} \approx 68 \text{ mph}$$

$$60^2 = x^2 + 40^2$$

$$\sqrt{60^2 - 40^2} = x$$

$$x \approx 44.$$

$144x^2 -$

(5)

$$y^2 = 100^2 + (100-x)^2$$

$$= 100^2 + 100^2 - 200x + x^2$$

$$= x^2 - 200x + 2 \cdot 100^2$$

$$y = \sqrt{x^2 - 200x + 20000}$$

$2y\frac{dy}{}$

$$T(x,y) = \frac{x}{40} + \frac{y}{6}$$

$$T(x) = \frac{1}{40}x + \frac{1}{6}\sqrt{x^2 - 200x + 20000}$$

domain $x^2 - 200x + 20000 > 0$

$$T'(x) = \frac{1}{40} + \frac{1}{12}\frac{(2x-200)}{\sqrt{x^2-200x+20000}} = 0$$

$$\frac{1}{40} = \frac{2x-200}{12\sqrt{\quad}}$$

$$12\sqrt{\quad} = 40(2x-200)$$

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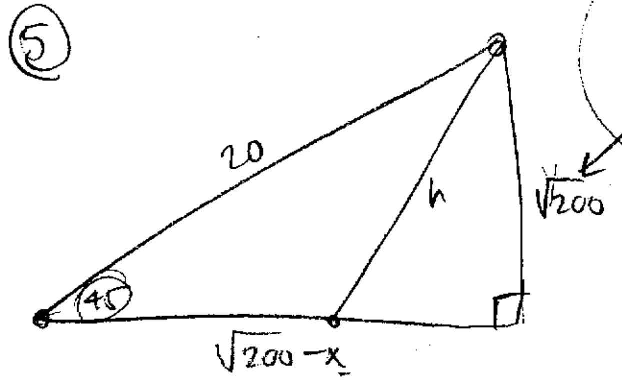

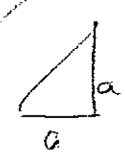

$2a^2 = 20^2$

$a^2 = 200$

$a = \sqrt{200}$

$h^2 = x^2 + 200 \qquad h = \sqrt{x^2+200}$

$T(x,y) = \frac{1}{40}(\sqrt{200} - x) + \frac{1}{6}h.$

$T(x) = \frac{1}{40}(\sqrt{200} - x) + \frac{1}{6}\sqrt{x^2+200}$

$T'(x) = \frac{1}{40} + \frac{1 \cdot \cancel{2}x}{\cancel{12}\,6\sqrt{x^2+200}} = 0$

$\frac{x}{6\sqrt{x^2+200}} = -\frac{1}{40}$

$\frac{x^2}{36(x^2+200)} = \frac{1}{1600}$

$1600x^2 = 36x^2 + 7200$

$1564x^2 = 7200$

$x = \sqrt{\frac{7200}{1564}}$

$x \approx 2.155$

he should run

$\sqrt{200} - 2.155$

$= 11.99$ feet down the beach

6 ~~5~~. (20 points) Sketch the graph of the function on the following page. Use this page do do your work. Include all relevant features and label them on the graph. Organize your work clearly.

$$f(x) = \frac{x-1}{x^2}$$

domain $\{x \in \mathbb{R};\ x \neq 0\}$

Intercepts: $f(0)$ undefined ← no y-int

$\frac{x-1}{x^2} = 0$

so $x - 1 = 0$

$x = 1$ ← x-int.

sign analysis for $f(x)$

	$(-\infty, 0)$	0	$(0,1)$	1	$(1, \infty)$
test	-1		$\frac{1}{2}$		2
$f(x)$	$-$		$-$		$+$
graph	below		below		above

asymptotes

- $\lim_{x\to\pm\infty} \frac{x-1}{x^2} = 0$ so $y=0$ is a HA
- $x=0$ is a V.A.
  behavior: • $\lim_{x\to 0^+} \frac{x-1}{x^2} = -\infty$
  • $\lim_{x\to 0^-} \frac{x-1}{x^2} = -\infty$

critical points

$f'(x) = \frac{x^2 - (x-1)\cdot 2x}{x^4}$ domain $\{x \in \mathbb{R} \mid x \neq 0\}$.

$= \frac{x^2 - 2x^2 + 2x}{x^4}$ no type 1 crit pts.

$= \frac{2x - x^2}{x^4}$

type 2: $\frac{2x - x^2}{x^4} = 0$ if $2x - x^2 = 0$

$x(2-x) = 0$

not in domain → $x = 0$ $x = 2$, $f(2) = \frac{1}{4}$

sign analysis for $f'(x)$:

	$(-\infty, 0)$	0	$(0,2)$	2	$(2, \infty)$
test	-1		1		10
$f'(x)$	$-$		$+$		$-$
graph	↘		↗		↘

local max @ $(2, \frac{1}{4})$.

$f(x)$ is increasing on $(0,2)$
decreasing on $(-\infty, 0) \cup (2, \infty)$

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sign analysis for $f''(x)$:

$f''(x) = \frac{(2-2x)(x^4) - (2x - x^2)(4x^3)}{x^8}$

$= \frac{2x^4 - 2x^5 - 8x^4 + 4x^5}{x^8}$

$= \frac{2x^5 - 6x^4}{x^8}$

$f''(x) = 0$ if $\frac{2x^5 - 6x^4}{x^8} = 0$

so $2x^4(x-3) = 0$

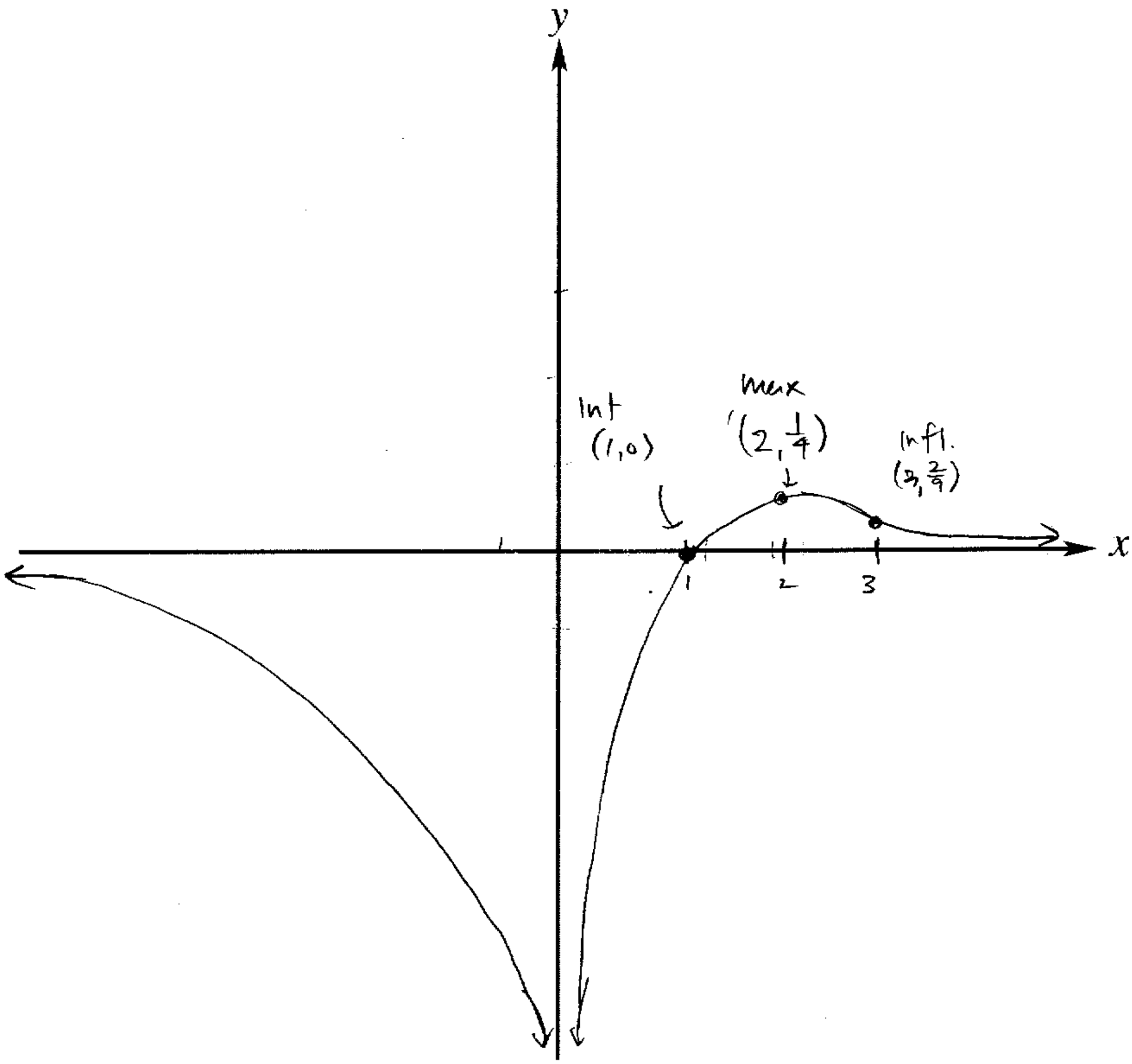
not in domain → $x=0$ $x = 3$, $f(3) = \frac{2}{9}$

sign analysis for $f''(x)$:

	$(-\infty, 0)$	0	$(0,3)$	3	$(3, \infty)$
test	-10		1		10
$f''(x)$	$-$		$-$		$+$
graph	$\cap$		$\cap$		$\cup$

point of inflection at $(3, \frac{2}{9})$

$f(x)$ is concave up on $(3, \infty)$
down $(-\infty, 0) \cup (0, 3)$



3g.