MAT 1475H **Name (Print):** ____________________
Spring 2014
Professor K. Poirier
Test #1
March 12, 2014 **Time Limit: 100 Minutes**

This exam contains 10 pages and 9 problems. Check to see if any pages are missing. Print your name on the top of this page, and put your initials on the top of every page, in case the pages become separated.

You may use a calculator on this test. No other aids are allowed.

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1. Evaluate the following limits $\lim_{x\to a} f(x)$. For each, use your answer to describe the behavior of the function $f(x)$ near the point $x = a$. (That just means use words to describe the graph.)

   (a) (5 points)

$$\lim_{x\to 0} \frac{4^{2x} - 1}{4^x - 1}$$

   (b) (5 points)

$$\lim_{x\to 0} \frac{\sin(5x)}{\sin(3x)}$$

(c) (5 points)

$$\lim_{x \to 4^+} \frac{x}{x^2 - 4x}$$

(d) (5 points)

$$\lim_{x \to -\infty} \frac{x}{x^2 - 4x}$$

2. (a) (5 points) Use the limit definition of the derivative to differentiate:

$$f(x) = x^3 - x^2$$

(b) (5 points) Determine the equation of the line tangent to the graph of $f(x) = x^3 - x^2$ at the point where $x = -1$. Leave your answer in $y = mx + b$ form.

3. Let $f(x) = x^5 + x^4 + x^3 + x^2 + x + 1$.

   (a) (3 points) Find $f'(x)$.

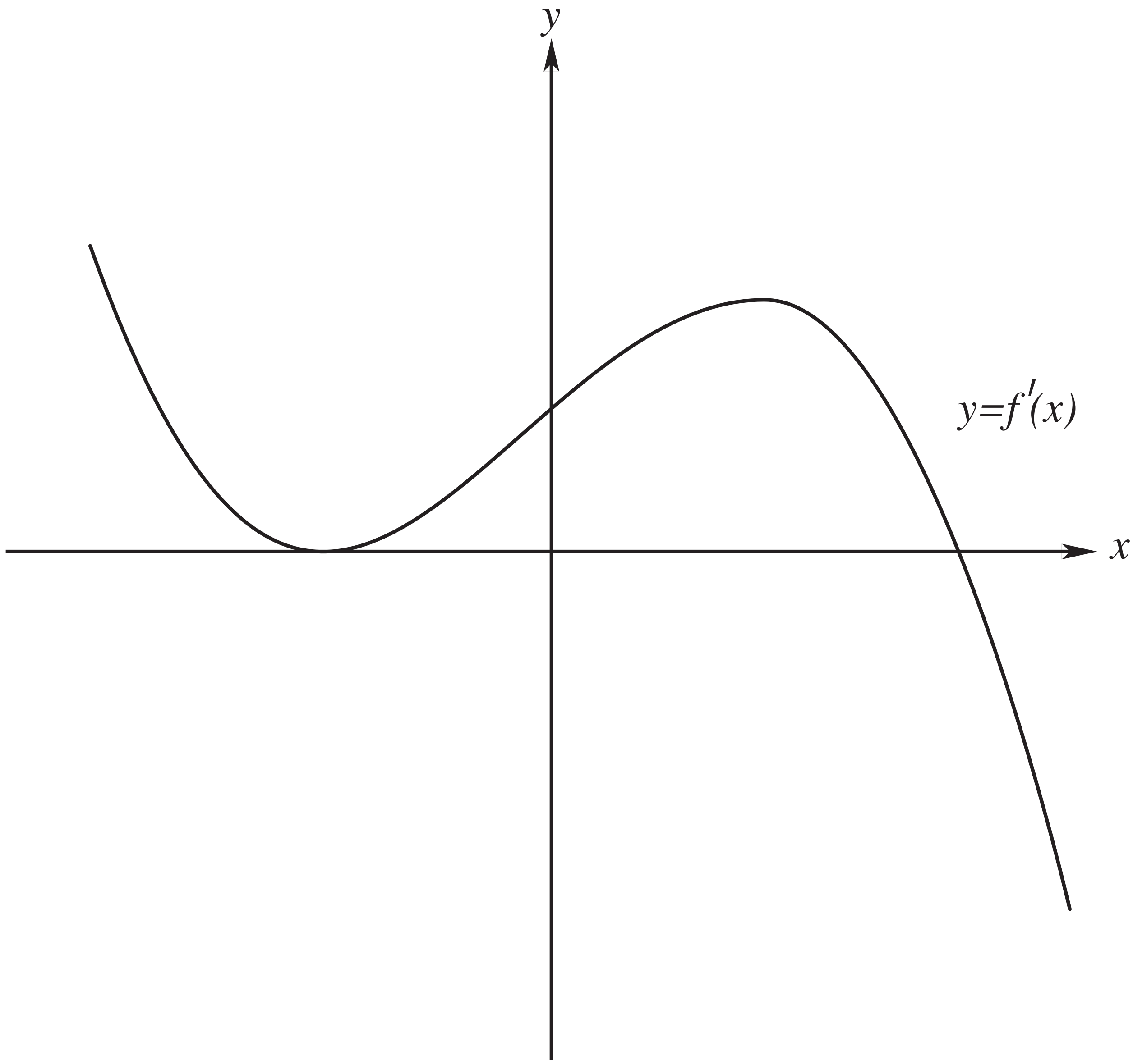
   (b) (2 points) Is $f(x)$ the derivative of another function $g(x)$? If so, find $g(x)$ so that $g'(x) = f(x)$.

4. (10 points) Differentiate; do not simplify your answer”

$$f(x) = \left(\frac{e^{4x} + x}{\tan(x) + 2^x}\right)^5 \cdot \sqrt{\frac{\ln(x) + x^2}{\sin^{-1}(3x) - 1}}$$

Hint: This is a long one. It might be easier to break this down into smaller pieces, differentiate those, and then assemble the pieces to form your answer. Partial credit will be awarded for the individual pieces—as long as it’s clear what you’re doing—even if they’re not assembled correctly in the end.

5. (a) (3 points) Use the following graph of $f'(x)$ to graph a function $f(x)$.



(b) (2 points) Is there more than one possible graph for $f(x)$? Explain.

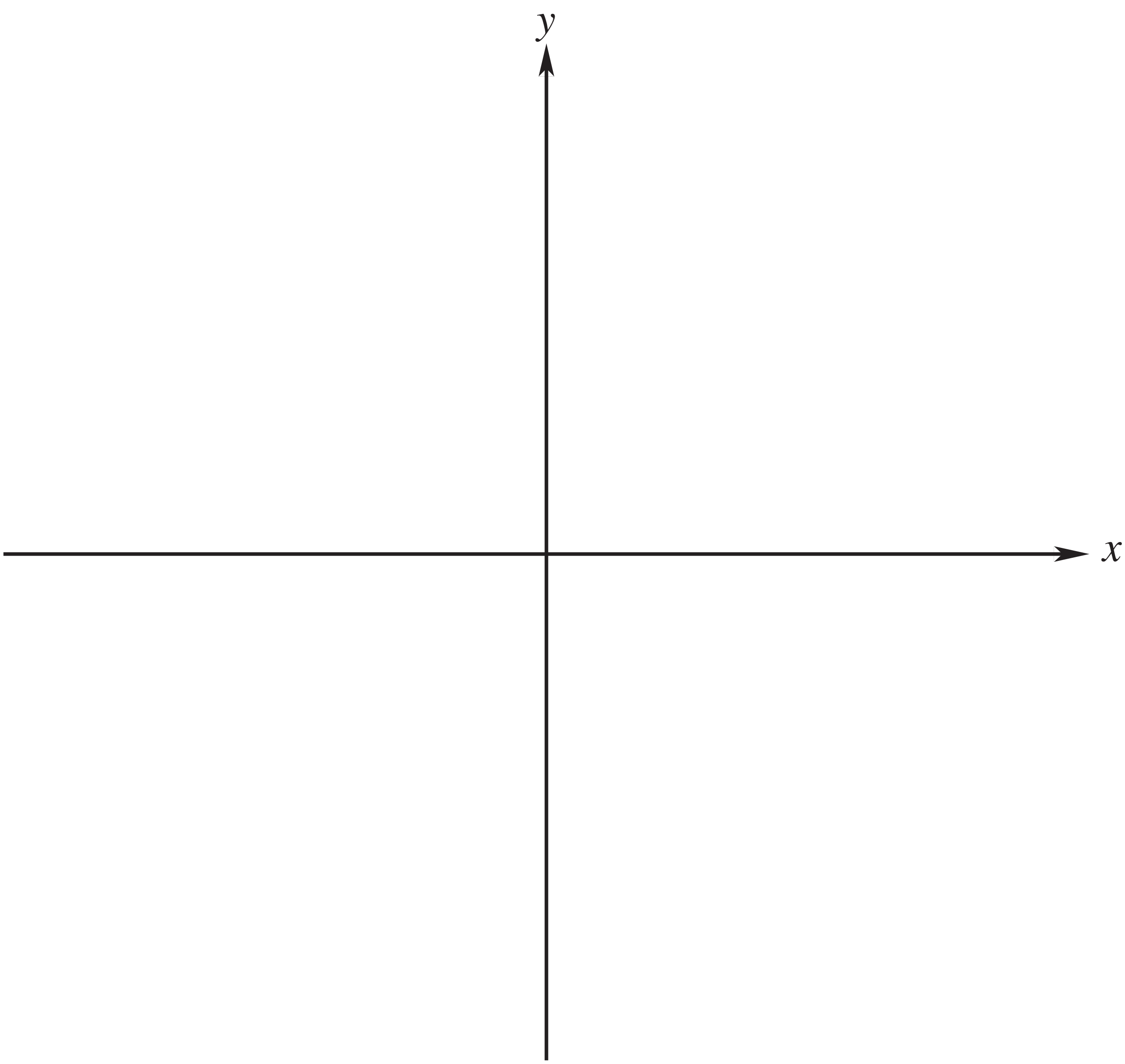
6.

$$f(x) = \begin{cases} x^3 \sin(\frac{1}{x}), & x \neq 0 \\ 0, & x = 0 \end{cases}$$

(a) (3 points) Is $f(x)$ continuous at $x = 0$? Explain.

(b) (2 points) Is $f(x)$ differentiable at $x = 0$? If so, determine $f'(0)$.

7. (10 points) Graph a single function $f(x)$ satisfying all of the following conditions.
   (a) The domain of $f(x)$ is $\mathbb{R}$.
   (b) $f(0) = 1$
   (c) $\lim_{x \to 0^-} f(x) = -1$
   (d) $\lim_{x \to 0^+} f(x) = -\infty$
   (e) $\lim_{x \to \infty} = 1$
   (f) $f'(1) = 2$
   (g) $f'(2) = 0$
   (h) $f(x)$ is not differentiable at $x = -1$.



8. Determine whether each of the following statements are true or false. You do not need to justify your answer.

   (a) (1 point) If $\lim_{x\to a} f(x)$ exists, then $f(x)$ is continuous at $x = a$.

   Circle one: true false

   (b) (1 point) If $\lim_{x\to a} \frac{f(x)-f(a)}{x-a}$ exists, then $f(x)$ is differentiable at $x = a$.

   Circle one: true false

   (c) (1 point) If $f(x)$ is differentiable at $x = a$, then $f(x)$ is continuous at $x = a$.

   Circle one: true false

   (d) (1 point) If $\lim_{x\to a} f(x)$ exists, then $\lim_{x\to a^+} f(x)$ exists.
   Circle one: true false

   (e) (1 point) If $f(a)$ exists, then $\lim_{x\to a} f(x)$ exists.

   Circle one: true false

9. EXTRA CREDIT (This question will be graded stringently.)

   (a) (1 point) State the ε-δ definition of limit

   $$\lim_{x\to a} f(x) = L.$$

   (b) (4 points) Use the $\varepsilon - \delta$ definition of limit to show

   $$\lim_{x\to 2} x^3 = 8.$$