MAT 1475 H HOMEWORK #5 SOLUTIONS

$\boxed{3.3 \ \#64}$

We'll show that c is a multiple root of $f(x)$ if and only if c is a root of $f(x)$ and $f'(x)$. This means that the two conditions are equivalent. We'll show that one condition implies the other, and vice versa.

- First, we'll assume that c is a multiple root of $f(x)$ and we'll show that c is a root of $f'(x)$.

  Since c is a multiple root of $f(x)$,
  $$f(x) = (x-c)^2 h(x).$$
  so $f'(x) = 2(x-c)\cdot h(x) + (x-c)^2 h'(x)$ by the product rule
  $$= (x-c)\left[2h(x) + (x-c)h'(x)\right].$$
  So $x-c$ is a factor of $f'(x)$
  and c is a root of $f(x)$ (which was what we wanted to show)

- Now, we'll assume that c is a root of $f(x)$ and $f'(x)$ show that c is a multiple root of $f(x)$.

  Since c is a root of $f(x)$, we know
  $$f(x) = (x-c)h(x)$$
  so $f'(x) = h(x) + (x-c)h'(x)$

  Since c is a root of $f'(x)$ we know
  $$f'(x) = (x-c)g(x) = h(x) + (x-c)h'(x)$$
  Since $(x-c)$ is a factor of the left side it is also a factor of the right hand side, so it's a factor of $h(x)$ and $f(x) = (x-c)(x-c)k(x)$ and c is a multiple root of $f(x)$

3.3 # 65

(a) $f(x) = x^5 + 2x^4 - 4x^3 - 8x^2 - x + 2$

$f'(x) = 5x^4 + 8x^3 - 12x^2 - 16x - 1$

$f'(-1) = 5(-1)^4 + 8(-1)^3 - 12(-1)^2 - 16(-1)$

$= 5 - 8 - 12 + 16$

$\neq 0$ so -1 is not a multiple root of $f(x)$

(b) $f(x) = x^4 + x^3 - 5x^2 - 3x + 2$

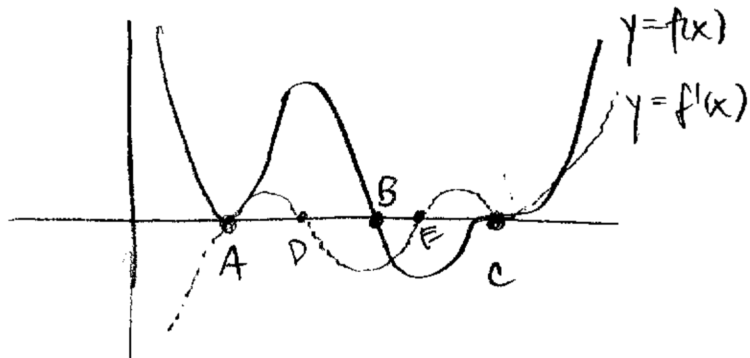
$f'(x) = 4x^3 + 3x^2 - 10x - 3$

$f'(-1) = 4(-1)^3 + 3(-1)^2 - 10(-1) - 3$

$= -4 + 3 + 10 - 3$

$\neq 0$ so -1 is not a multiple root of $f(x)$

3.3 #66



A quick sketch of the graph of $f'(x)$ shows it has roots at A, D, E, and C

f(x) has roots at A, B, and C

$\therefore$ f(x) has multiple roots at A and C

(Notice: f(x) has a multiple root — and f'(x) has a root — at places where the graph of f(x) has an x-intercept <u>and</u> the tangent there is horizontal)