HOMEWORK # 3 | SOLUTIONS

2.5 PRELIM Q #2

(a) $f(c)$ indeterminate Then $\lim_{x \to c^-} f(x) \neq \lim_{x \to c^+} f(x)$

counterexample: any function where $f(c)$ is undefined and $f(x)$ has a removable discontinuity at $x=c$,

e.g. $f(x) = \dfrac{(x-1)^2}{x-1}$, $c = 1$, $\lim_{x \to 1} f(x) = 0$

(b) If $\lim_{x \to c} f(x)$ exists, then $f(c)$ is not indeterminate

counterexample: any function where $f(c)$ is undefined and $f(x)$ has a removable discontinuity at $x=c$

e.g. $f(x) = \dfrac{(x-1)^2}{x-1}$, $c=1$, $\lim_{x \to 1} f(x) = 0$

(c) If $f(x)$ is undefined at $x=c$, then $f(x)$ has an indeterminate form at $x=c$

counterexample: any function that is undefined at $x=c$ and naive sub does not give an indeterminate form, like $\frac{0}{0}$ at $x=c$

e.g. $f(x) = \dfrac{1}{x-1}$, $c=1$.

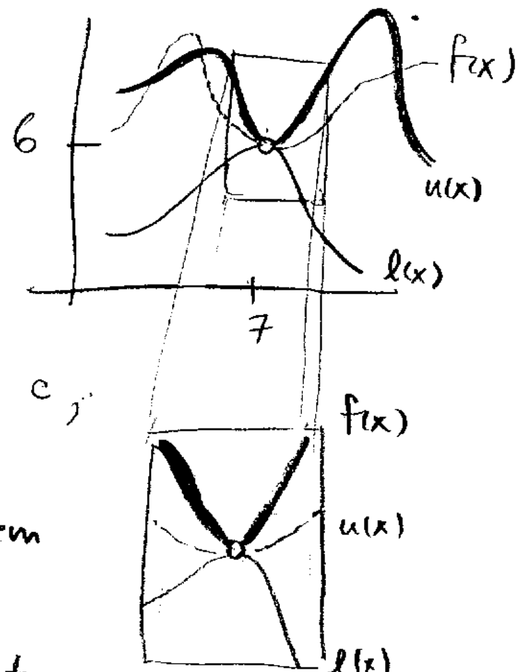
2.5 FURTHER INSIGHTS #55

$$\lim_{x\to c} \frac{x^2-5x-6}{x-c}$$

the limit exists if and only if $x-c$ is a factor of x^2-5x-6
$=(x-6)(x+1)$

$\therefore\ c=6$ or $c=-1$.

2.6 #3



Know: $\lim_{x\to 7} l(x) = \lim_{x\to 7} u(x) = 6$.

The squeeze theorem says "Assume for $x \neq c$, x in some interval containing c, $l(x) \le f(x) \le u(x)$ ....."

The point is that the squeeze theorem may be applied if you can zoom in enough so that the string of inequalities is satisfied. This is true in this case, so we can conclude that $\lim_{x\to 7} f(x) = 6$.

2.6 #33

$$\lim_{\theta\to 0} \frac{\sin(7\theta)}{\sin(3\theta)} = \lim_{\theta\to 0} \frac{7\theta}{7\theta}\cdot\frac{3\theta}{3\theta}\cdot\frac{\sin(7\theta)}{\sin(3\theta)}$$

$$= \lim_{\theta\to 0} \frac{7\theta}{3\theta}\cdot\frac{\sin(7\theta)}{7\theta}\cdot\frac{1}{\frac{\sin(3\theta)}{3\theta}}$$

as $\theta\to 0$ both $7\theta\to 0$ and $3\theta\to\theta$

$$= \frac{7}{3} \lim_{7\theta\to 0} \frac{\sin(7\theta)}{7\theta}\cdot\frac{1}{\lim_{3\theta\to 0}\frac{\sin(3\theta)}{3\theta}}$$

$$= \frac{7}{3}\cdot 1\cdot\frac{1}{1}$$

$$= \frac{7}{3}$$

$\boxed{2.6\ \#51}$

$$\lim_{\theta \to 0} \frac{1-\cos\theta}{\theta} = \lim_{\theta \to 0} \frac{1-\cos(\theta)}{\theta} \cdot \frac{1+\cos(\theta)}{1+\cos(\theta)}$$

$$= \lim_{\theta \to 0} \frac{1-\cos^2(\theta)}{\theta(1+\cos(\theta))}$$

$$= \lim_{\theta \to 0} \frac{\sin^2(\theta)}{\theta(1+\cos(\theta))}$$

$$= \lim_{\theta \to 0} \frac{\sin(\theta)}{\theta} \cdot \frac{\sin(\theta)}{1+\cos(\theta)}$$

$$= \lim_{\theta \to 0} \frac{\sin(\theta)}{\theta} \cdot \lim_{\theta \to 0} \frac{\sin(\theta)}{1+\cos(\theta)}$$

$$= 1 \cdot \frac{0}{2}$$

$$= 0.$$