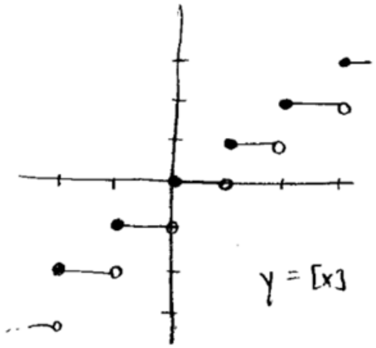
MAT 147CH HOMEWORK #2 SOLUTIONS.

2.2 #37



Let c be any integer.

(a) $\lim_{x \to c^-} [x] = c-1$ Here, x is close to c but $x < c$, so the greatest integer less than or equal to x is $c-1$

(b) $\lim_{x \to c^+} [x] = c$ Here, x is close to c but $x > c$, so the greatest integer less than or equal to x is c.

2.3 #31

The quotient law states that if

(1) $\lim_{x \to c} f(x)$ exists,

(2) $\lim_{x \to c} g(x)$ exists, and

(3) $\lim_{x \to c} g(x) \neq 0$

then $\lim_{x \to c} \frac{f(x)}{g(x)} = \frac{\lim_{x \to c} f(x)}{\lim_{x \to c} g(x)}$ (That is, you can take the limits of f(x) and g(x) separately, and then take the quotient.

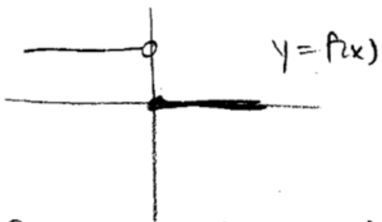
If $f(x) = \sin(x)$, $g(x) = x$, and $x = c$, then conditions (1) and (2) are satisfied, but (3) is not satisfied, so the quotient law may not be applied to evaluate $\lim_{x \to 0} \frac{\sin(x)}{x}$.

(Recall, we sketched a proof that $\lim_{x \to 0} \frac{\sin(x)}{x} = 1$ in class).

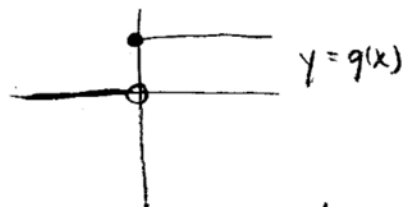
2.4 #81

If f(x) and g(x) are each discontinuous at c, this does <u>not</u> imply that f(x) + g(x) must be discontinuous at c. To show this, we need to come up with only one counterexample, though several exist. Here's one easy one:

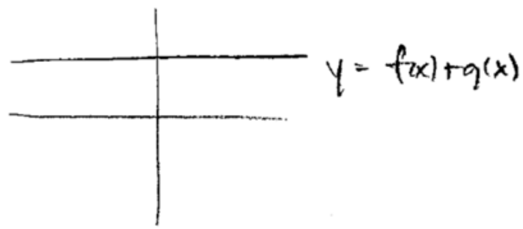
$$f(x) = \begin{cases} 0, & \text{if } x \geq 0 \\ 1, & \text{if } x < 0 \end{cases} \qquad g(x) = \begin{cases} 1, & \text{if } x \geq 0 \\ 0, & \text{if } x < 0 \end{cases}$$



f(x) is discontinuous at x = 0



g(x) is discontinuous at x = 0



f(x) + g(x) = 1 (for any x-value), which is <u>continuous</u> at x = 1.

This <u>doesn't</u> contradict Theorem 1 (i), which makes a statement about the continuity of f(x) + g(x) at x = c, <u>given</u> the continuity of f(x) and g(x) at x = c. Above, all we're saying is: f(x) and g(x) don't <u>have</u> to be continuous at x = c for f(x) + g(x) to be continuous at x = c. Theorem 1 (i) says that if you already know that f(x) and g(x) are continuous at x = c, then you know that f(x) + g(x) is too.

2.4 #84

CONTINUOUS

(a) velocity of an airplane

(b) temperature of a room

DISCONTINUOUS

(c) value of a bank account

(d) salary of a teacher

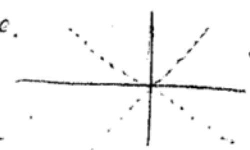
(e) population of the world.

(I think about the first column as "analog" quantities and the second column as "digital").

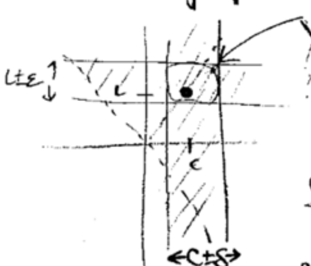
2.4 #88

$$f(x) = \begin{cases} x & \text{if } x \text{ rational} \\ -x & \text{if } x \text{ irrational} \end{cases}$$

The graph of this function looks something like.



While we haven't been using the formal definition of limit, it might be easiest to see how it fails here for all x-values except $x=0$. First, assume $x \neq 0$. Then near such a point on the graph of $f(x)$, there is an $\varepsilon > 0$ with no "good δ."



You can't find a δ so that the part of the graph inside the vertical band lies inside the horizontal band (the graph jumps up and down too much). So if $c \neq 0$, $\lim_{x \to c} f(x)$ can't exist, so $f(x)$ can't be continuous at $x = c$.

If $c = 0$, though, $\lim_{x \to 0} f(x) = 0$. ($\delta = \varepsilon$ is a "good δ").

0 is a rational number, so $f(0) = 0$.

Since $\lim_{x \to 0} f(x) = f(0)$, $f(x)$ is continuous at $x = 0$.