MAT 1475 TEST 3 SOLUTIONS

① y is not given in terms of x, so use implicit differentiation to find $\frac{dy}{dx}$

$$y^2-7xy+x^3-2x=9$$

$$2y\frac{dy}{dx}-7y-7x\frac{dy}{dx}+3x^2-2=0$$

at (0,3) $2\cdot 3\frac{dy}{dx}-7\cdot 3-7\cdot 0\cdot\frac{dy}{dx}+3\cdot 0^2-2=0$

$$6\frac{dy}{dx}-21 \qquad -2=0$$

$$6\frac{dy}{dx}=23$$

$$\frac{dy}{dx}=\frac{23}{6}$$

$$y-\underset{\underset{3}{\uparrow}}{y_1}=\underset{\underset{\frac{23}{6}}{\uparrow}}{m}(x-\underset{\underset{0}{\uparrow}}{x_1}) \longrightarrow y-3=\frac{23}{6}(x-0)$$

$$y=\frac{23}{6}x+3$$

② $f(x)=\sqrt{x}$

$a=16$

$f(16)=\sqrt{16}$
$=4$

$f'(x)=\frac{1}{2\sqrt{x}}$

$f'(16)=\frac{1}{2\sqrt{16}}$
$=\frac{1}{8}$

linearization of $f(x)=\sqrt{x}$ at $a=16$:

$$L(x)=\underset{\underset{\frac{1}{8}}{\uparrow}}{f'(a)}(x-\underset{\underset{16}{\uparrow}}{a})+\underset{\underset{4}{\uparrow}}{f(a)}$$

$$L(x)=\frac{1}{8}(x-16)+4$$

$$L(16.4)=\frac{1}{8}(16.4-16)+4$$
$$=\frac{1}{8}(0.4)+4$$
$$=0.05+4$$
$$=4.05\approx\sqrt{16.4}$$

③ $f(x)=x^3-3x^2-1$ domain $=\mathbb{R}$

$f'(x)=3x^2-6x$ domain $=\mathbb{R}$ (no type 2 critical pts)

critical pts of type 1:

solve $3x^2-6x=0$

$3x(x-2)=0$

$\boxed{x=0}$ not in [1,4] so ignore

$x=2$

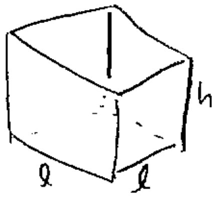
$f(1)=1^3-3\cdot1^2-1$
$=-3$

$f(2)=2^3-3\cdot2^2-1$
$\boxed{=-5}$ ← min value

$f(4)=4^3-3\cdot4^2-1$
$\boxed{=15}$ ← max value

④ $V=256\ in^3$:

minimize surface area



$SA=\ell^2+4\ell \boxed{h}$

want to solve for one variable in terms of the other; use

$$256=\ell^2h$$

$$\frac{256}{\ell^2}=\boxed{h}$$

so $SA(\ell)=\ell^2+4\ell\cdot\left(\frac{256}{\ell^2}\right)$ $\ell>0$

$$=\ell^2+1024\ell^{-1}$$

$SA'(\ell)=2\ell-1024\ell^{-2}$ $\ell>0$

no type 2 critical pts

type 1: solve $2\ell-\frac{1024}{\ell^2}=0$

$$2\ell=\frac{1024}{\ell^2}$$

$$\ell^3=512$$

$$\ell=8$$

also need $h=\frac{256}{\ell^2}=\frac{256}{8^2}=4$

⑤ $\lim_{x\to0}\frac{e^{-10x}-e^{-5x}}{25x}$ type: $\frac{0}{0}$ apply l'Hôpital's rule

$=\lim_{x\to0}\frac{-10e^{-10x}+5e^{-5x}}{25}$ ← differentiate numerator ← differentiate denominator

$$=\frac{-10e^0+5e^0}{25}$$

$$=\frac{-5}{25}$$

$$=-\frac{1}{5}$$

⑥ $f(x)=\frac{x}{e^{3x}}$

(a) domain of $g(x)=e^{3x}$ is $\mathbb{R}$ and range of $g(x)=e^{3x}$ is $(0,\infty)$

in particular, e^{3x} is never zero for any value of x

so domain of $f(x)=\frac{x}{e^{3x}}$ is $\mathbb{R}$

(b) f(x) has no discontinuities

(c) y intercept $f(0)=\frac{0}{1}=0$, which also tells us 0 is the only x-int

⑥ cont

(d) no vertical asymptotes since there's no value a for which $\lim_{x\to a^{\pm}} \frac{x}{e^{3x}} = \pm\infty$.

horizontal asymptotes:

- $\lim_{x\to\infty} \frac{x}{e^{3x}}$ ← limit of type $\frac{\infty}{\infty}$ apply l'Hopital's rule

$= \lim_{x\to\infty} \frac{1}{3e^{3x}}$ ← as x gets large $3e^{3x}$ gets large

$= 0$

so $y=0$ is a H.A. (on the right)

- $\lim_{x\to-\infty} \frac{x}{e^{3x}}$ x ← approaches $-\infty$; e^{3x} ← positive, approaches 0

$= -\infty$ so no H.A. on the left.

(e) $f'(x) = \frac{e^{3x} - 3xe^{3x}}{e^{6x}}$ domain: ℝ so no type 2 critical pts.

type 1 critical pts: solve

$$\frac{e^{3x} - 3xe^{3x}}{e^{6x}} = 0$$

$$e^{3x} - 3xe^{3x} = 0$$

$$e^{3x}(1-3x) = 0$$

↓ never 0

↓ $1-3x=0$, $x = \frac{1}{3}$

	$(-\infty,\frac{1}{3})$	$\frac{1}{3}$	$(\frac{1}{3},\infty)$
test pt	0		1
f'(test pt)	$f'(0) = \frac{e^0 - 3\cdot 0 e^0}{e^{6\cdot 0}}$ $= \frac{1-0}{1}$ $= 1 > 0$		$f'(1) = \frac{e^{3\cdot 1} - 3\cdot 1\cdot e^{3\cdot 1}}{e^{6\cdot 1}}$ $= \frac{e^3 - 3e^3}{e^6}$ $= -\frac{2e^3}{e^6}$ $= -\frac{2}{e^3} < 0$
f(x)	↗		↘

f(x) is increasing on $(-\infty,\frac{1}{3})$, decreasing on $(\frac{1}{3},\infty)$

(f) We can use our work from (e) to apply the first derivative test: there is a local max @ $x = \frac{1}{3}$

y coord: $f(\frac{1}{3}) = \frac{\frac{1}{3}}{e^{3\cdot\frac{1}{3}}} = \frac{1}{3e} \approx 0.906$

(g) simplify f'(x) first:

$$f'(x) = \frac{e^{3x}(1-3x)}{e^{6x}} = \frac{1-3x}{e^{3x}}$$

$$f''(x) = \frac{-3e^{3x} - (1-3x)3e^{3x}}{e^{6x}}$$

domain = ℝ.

solve $\frac{-3e^{3x} - 3(1-3x)e^{3x}}{e^{6x}} = 0$

$$-3e^{3x}(1 + (1-3x)) = 0$$

↓ never 0

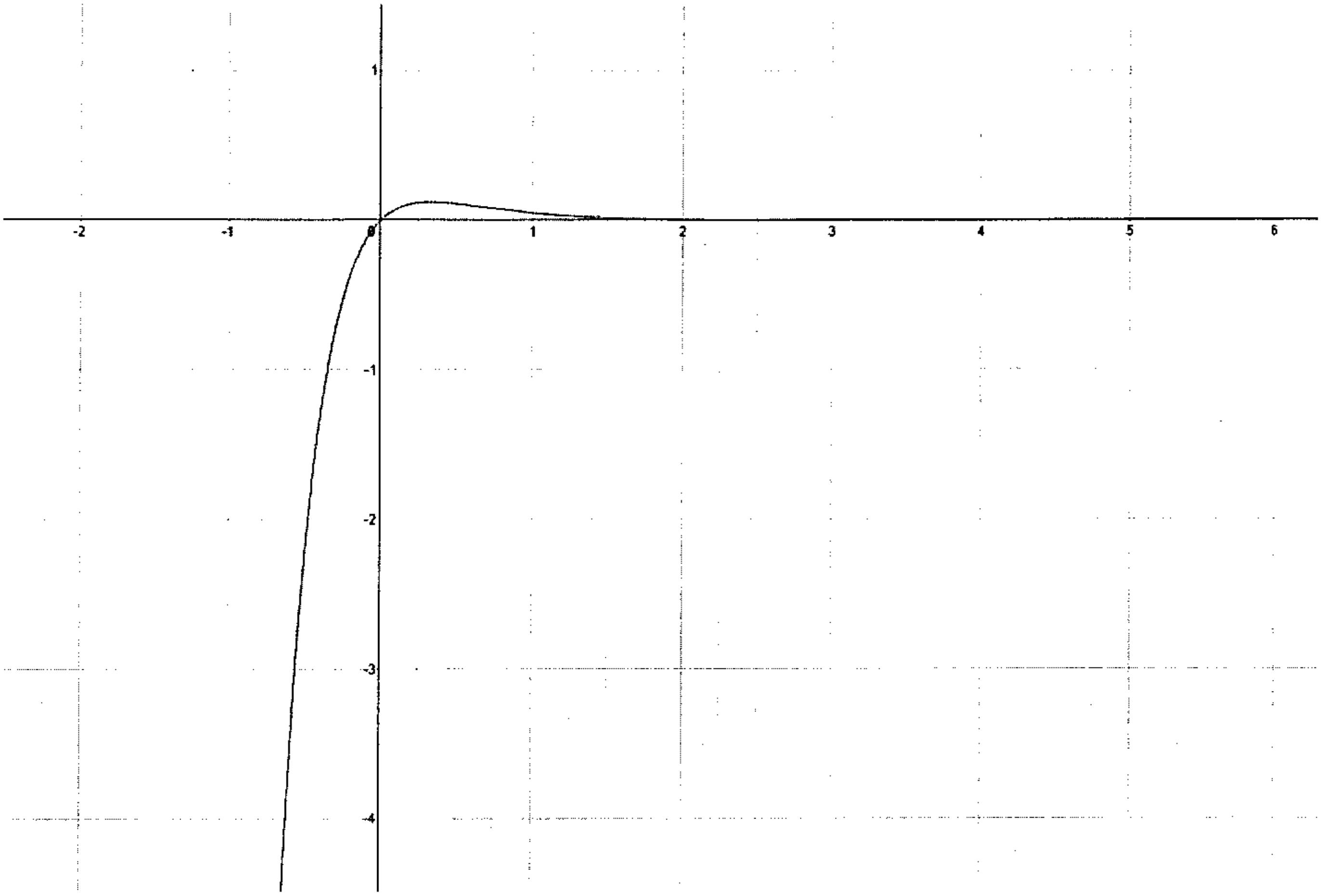
↓ $2-3x=0$, $x = \frac{2}{3}$

	$(-\infty,\frac{2}{3})$	$\frac{2}{3}$	$(\frac{2}{3},\infty)$
test	0		1
f''(test)	$f''(0) = \frac{-3-(1-0)\cdot 3}{1}$ $= \frac{-6}{1} < 0$		$f''(1) = \frac{-3e^3 - (1-3)\cdot 3\cdot e^3}{e^6}$ $= \frac{-3e^3(1+(-2))}{e^6}$ $= \frac{3e^3}{e^6} > 0$
f(x)	⌢		⌣

f(x) is concave down on $(-\infty,\frac{2}{3})$ and concave up on $(\frac{2}{3},\infty)$

(h) our work in (g) shows us that concavity changes at $x = \frac{2}{3}$

y coordinate $f(\frac{2}{3}) = \frac{\frac{2}{3}}{e^{3\frac{2}{3}}} = \frac{2}{3e^2} \approx 0.9022$


1
-1
-2
-3
-4
-2
-1
0
1
2
3
4
5
6